Analysis of biophysical processes with regard to advantages and disadvantages of irrigation mosaics

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2. Executive Summary

Irrigation mosaics are irrigation schemes in which small patches of irrigation occur within a region rather than irrigation of one large contiguous area. This report investigates methods for analysing irrigation mosaics in terms of their biophysical effects and impacts compared with a large contiguous area of irrigation. To estimate the effect of irrigation size and impact a scaling method was developed which calculates the marginal impact of having mosaics compared to one large contiguous area. We are able to show that by using a power function for the scaling that only one parameter is required to determine whether irrigation mosaics will result in positive, neutral or negative effects on the environment for a particular property of the irrigation system, compared with one big contiguous patch of irrigation of equal area. The various properties examined in this report are compiled into a summary table (Table 3). Given our current understanding there are both positive and negative environmental effects associated with irrigation mosaics. The weighting given to each of these environmental aspects is beyond the scope of this study, but when a weighting is assigned with the appropriate positive, zero, or negative sign it will provide a means to assist with analysis and management of mosaics.

In order to examine the effects of increased recharge due to irrigation on water table rise under irrigation patches a new mathematical solution was derived for steady recharge to a water table from circular patches. This was required to reduce the computational difficulties associated with existing mathematical solutions. This solution is for an infinite (unbounded) aquifer with no leakage through the base of the aquifer. The solution has great utility as it is developed in non-dimensional variables, which greatly reduces computational cost. The results show that the water table rise under an irrigation patch is strongly dependent on the size (radius) of the patch. It also indicates that the water table rise is linear with time until the system starts to come into an approximate steady-state with the surrounding area (note that no true steady-state is possible unless discharge to another water body occurs and the domain is fully bounded). For example the water table rise after 100 years for a 100 m radius patch with 1 mm/day of recharge is 3.65 m, while for a 100 km patch this is 365 m. This part of the work was for single isolated patches assuming idealised conditions.

A scheme for square and hexagonal centred regular grids of irrigation patches was developed using the principle of superpositioning. These allowed the examination of spacing and size of irrigation patches. The results show that as the spacing increases the mosaic grids water table rise tends towards that of an isolated patch. Also as the non-dimensional time, \( \tau = \bar{b}K/(S_yR^2) \) where \( R \) is the radius of the patch, \( t \) is time, \( \bar{b} \) is a linearisation parameter, \( K \) is the hydraulic conductivity of the aquifer and \( S_y \) is the specific yield) increases the distance between the patches to approximate an isolated patch increases. The mathematical approaches developed here will be useful in early stages of irrigation planning to help assess the likely number and spacing of irrigation patches that would make sense for particular landscapes.

Numerical solutions for irrigation mosaics were also used as they offer more reality and flexibility in material properties, boundary conditions and recharge rates than can be used in the analytical solutions. These numerical analyses show the same basic results as for the analytical solutions even though we used periodic rather than steady-state recharge in the analyses. The resulting periodic solutions show that the water table rises more slowly than it would with a steady-state recharge, but the same inevitable rise in water table height will occur. This has implications for designing and managing monitoring systems as the frequency of measurement will need to be high enough to capture the variable water table behaviour.
Solute transport was also examined in both the analytical and numerical modelling. The analytical modelling could only determine bounds for advected, non-reactive solutes. The results show that the radius impacted by solutes increases as the radius of the patch increases and the concentration becomes lower. Also, as the water table is lower with smaller isolated patches the salinisation potential could be greatly reduced if the initial water table height was well below the root zone.

Isolated irrigated patches have the potential to evaporate more water to the atmosphere due to advective effects caused by heat coming from surrounding dry areas. While there is no good research on this topic we have used results from studies on water bodies to infer what may occur for a well watered isolated patch. The results indicate that a 100 m radius patch may evaporate 28% more water than a 100 km radius patch. However, as we add more patches into the landscape it is likely that the evaporation rate will approach that of the large contiguous irrigated area due to entrainment of the water vapour by the convective boundary layer.

We have also examined the difference in the climatic conditions in northern Australia compared with southern Australian irrigated areas. Three locations in each climatic zone were selected and the rainfall, potential evaporation and water balance calculated. The results show that the Southern sites have relatively even rainfall through the year while the Northern sites have a distinctive wet season. For potential evaporation it is the Southern sites that show more annual variation, while the Northern sites show much less annual variation and much higher potential evaporation rates all year. The resulting water balances show almost continuous potential water deficit for the Southern sites with water surplus only predicted occasionally in winter. The potential water balances are more varied for the Northern sites with water deficits occurring consistently throughout the dry season, but surpluses generally shown for the wet season.

This report has highlighted research gaps and further model development that is required to help with the further analysis of advantages and disadvantages and design of potential irrigation mosaics for northern Australia. These are:

1. development of analytical solutions for periodic recharge of ground water
2. analysis of solute transport using particle tracing methods
3. analysis of finite arrays of mosaics
4. investigation of the effect of advection on evaporation as effected by the size and number of irrigated patches
5. investigation of water and solute (particularly salts and nutrients) balance and irrigation requirements of sites across northern Australia.
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3. Introduction

Irrigation mosaics offer the opportunity to use irrigation in a patchwork manner within an area rather than irrigate the whole area. Such mosaic patterns may provide a means to better utilise the natural resources in a region such as water, soils, geomorphology, microclimate etc while minimising unwanted environmental impacts. In an earlier report (Paydar et al. 2007) we have summarised the results of a literature review on what information is available and applicable to irrigation mosaics.

England (1963) stated “Water-table and salinity problems are to a large degree disabilities of scale; to this degree isolated small or individual irrigation projects are free of them”. Here we develop some tools that will help to determine how small and how isolated irrigated areas need to be to minimise (and hopefully avoid) water table rise and salinity problems. To do this we explore scaling and various other modelling approaches to determine the likely biophysical effects of irrigation mosaics. We have developed some new ideas and models and made use of existing models where appropriate to do this. The area of irrigation mosaics as was pointed out in the previous report (Paydar et al. 2007) is an area where there is not much existing literature. Here we have focussed on developing a coherent process which will allow an objective assessment of the advantages and disadvantages of irrigation mosaics, at least from a biophysical point of view.

4. Principles and key processes

Irrigation has in recent history been developed around a scheme level often covering many square kilometres in extent. These schemes were often developed without much thought to the negative impacts of such a large change to hydrology, geochemistry and the general environment. This has lead to problems associated with groundwater rise and drainage, salinisation, loss of biodiversity and even economic ruin (Pearce, 2006). Irrigation mosaics may allow the development of smaller patches of irrigated agriculture developed throughout a region where by careful design and management the economic benefits are not outweighed by the environmental impacts (Fig. 1.)

Figure 1. Schematic diagram showing a) traditional large scale contiguous area of irrigation and b) smaller distributed patches of irrigation making up an irrigation mosaic.

Irrigation mosaics offer the opportunity to avoid irrigation of difficult (sodic, saline or poorly drained) soils or hydrologically and environmentally sensitive regions. However, we need to know what the effects of adopting mosaics hydrologically are likely to be. To assist with this
task we have developed various tools that can help us in understanding the advantages and disadvantages of irrigation mosaics.

4.1. Irrigation mosaics

4.1.1. Basic description of mosaics

Irrigation mosaics could consist of any non-contiguous arrangement of irrigated patches within the landscape. They could be of any shape and/or size but will generally be rectangular, or elliptical in shape. The simplest shape to work with mathematically is a circle that can merely be described by its radius, \( R \). We will use the circle in this report as the results are applicable to all other shapes when scaled correctly.

The approach taken will be to examine the marginal effects of size (radius) on a property or processes, associated with changes to the property or processes induced by irrigation. We consider the effect of irrigation on a property or processes to consist of a an effect which is constant with the size of the patch and the marginal effect which changes with the size. For example if evapotranspiration does not change with size of the patch then there is no marginal effect, whereas if the size of the patch does effect evaporation then the evaporation from the patch is given by a constant part and a marginal part. The approach taken is to examine the effect of the size of the patch on marginal effects for single patches and then to examine the effect of regular arrays of uniform sized patches. Since the effects of irrigation within the patch are likely to be the same it is these marginal effects which will determine the advantage or disadvantage of irrigation mosaics (see section 2.1.3 below). The methods described here can be used for any arrangement of patch sizes and array patterns by using non-dimensional methods we provided tools that reduce the computational effort required to investigate mosaics systems that reduce the likelihood of redundant computations.

4.1.2. Patches, distribution and connectivity

Two arrays of patches will be used in this report. These are a square grid and a hexagonal centred grid (Fig. 2). These arrays allow us to examine the effect of the spacing within the grid and the radius of the patches to be in relation to any marginal effects.
Figure 2. Grids for looking at irrigation mosaics a) square grid and b) hexagonal centred grid. The lines indicated for calculation of superpositioning will be discussed below. The grids extend to infinity within the 2-dimensional (2-D) plain.

The connectivity of the patches will depend on the spacing \( L \) and size of the patch \( R \). When the patches are far enough apart (this will be discussed more specifically below for water tables) then they may not be connected but act as individual patches. As they move closer together then there will be more connectivity between them until when \( L = 2R \) they are fully connected and are effectively one contiguous area. The connectivity will be some function of \( L/R \) but will vary possibly with time and the process or property being assessed. For example the ground water mound induced by recharge will grow with time. Initial for \( L/R > 2 \) there will be not overlap in the ground water mounds but as time increases overlap can occur. Also the connectivity may have lags in the system such as the response of the water table to the advent of irrigation. When irrigation is initiated and recharge to ground water increases it takes some time for the extra recharge to be transmitted through the vadose zone. This means that an increase in water table height is not detected and irrigation could be thought to not be having an effect on groundwater. However, this is not correct, as the effect is just delayed by the transmission time through the vadose zone.

The depth to the water table will have an effect on the response time of the aquifer to land management change, but in the calculations of water table response presented below this is not taken into account. However, the travel time \( t_w \) can be calculated from the base of the root zone to the water table by:

\[
t_w = \frac{(z_{WT} - z_{RT})\Delta \theta}{D_t}.
\]

where \( z_{WT} \) and \( z_{RT} \) are the depth to the water table and of the root zone, respectively [L], \( \Delta \theta \) is the change in water content of the vadoze zone caused by the change in the deep percolation rate to \( D_t \) [L T\(^{-1}\)]. Solute transport will be further delayed in its arrival at the water table and can be estimated by arrival of the advected (or piston front):
\[ t_s = \frac{R_a(z_{WT} - z_{RT})\theta_t}{D_t} \]  \[ [2] \]

Where \( t_s \) is the time for travel of solute from root zone to groundwater, \( \theta_t \) is the water content in the vadose zone behind the wetting front and \( R_a \) is the retardation factor for solutes that are retarded by chemical, biological or physical processes. For a non-retarded solutes \( R_a = 1 \).

### 4.1.3. Scaling framework

Here a scaling framework based on elliptical patches is developed which will allow the marginal effects of irrigation mosaics to be assessed. An elliptical framework is adopted as it can approximate a rectangular patch and in its simplest form becomes a circle.

![Ellipse with characteristic major axis, a and b.](image)

We will assume homogeneous conditions occur throughout the region and that the marginal costs or benefits are related to the size of the patch. Schematically this is presented in Figure 1, but the property and marginal value does not necessarily have to be the physical area so long as the property and marginal effects scale with area. Under these conditions we can characterise the system in terms of some length scale that allows us to examine the effects of the size of the mosaics on the hydrological or other properties. We can consider the characteristics of the spatial extent as being described approximately by an ellipse (Figure 1 and 3). The perimeter \( (P) \) and area \( (A) \) of an ellipse are given by:

\[ P = 4a \int_0^{\infty} \sqrt{1 - \left( \frac{a^2 - b^2}{a^2} \right) \sin^2 \theta} \, d\theta \]
\[ \approx 2\pi \sqrt{a^2 + b^2} \]  \[ [3] \]

\[ A = \pi ab \]

In a mosaic we will have patches spread throughout a region and the total area will then be the sum of each patch. If an irrigation scheme was implemented such that rather than one
contiguous irrigated area, it consisted of a number of smaller patches, we consider this to be and irrigation mosaic.

We assume that some property of the system, \( f \), scales with the area of the patch such that:

\[
f = C + \alpha_a a^{\beta_a} \alpha_b b^{\beta_b}
\]  

[4]

where \( C \) is the property that does not change with the area, \( a \) the minor axis, \( b \) the major axis of an ellipse, \( \alpha_a \) and \( \beta_a \) are empirical coefficients associated with the marginal impact of size on the property and \( x = a \) or \( b \). We define the impact of this property as:

\[
I(a,b) = \pi a b \left( C + \alpha_a a^{\beta_a} a^{\beta_b} \right) = \pi a b \left( C + \gamma a^{\beta_b} a^{\beta_b} \right)
\]  

[5]

where \( \gamma = \alpha_a a b_{\beta_b} \). For a system such as irrigation mosaics we wish to determine the effect of isolated distributed patches compared to one large contiguous patch. If the total area is the same for one contiguous area and \( n \) number of smaller patches of the same size then:

\[
\sum_{i=1}^{n} \pi a_i b_i = n \pi a b = \pi A B
\]

\[
n = \frac{A B}{a b}
\]  

[6]

We define the relative marginal impact of \( n \) patches compared to one contiguous area as:

\[
\frac{nI(a,b)}{I(A,B)} = I_{re} = \frac{n \gamma (a^{\beta_a} b^{\beta_b})}{\gamma (A^{\beta_b} B^{\beta_b})}
\]  

[7]

Simplification of eqn [7] can be achieved by assuming that \( \beta_a = \beta_b = \beta \) and with substitution of eqn [6] results in:

\[
I_{re} = \frac{n \gamma (ab)^{\beta}}{\gamma n^\beta (ab)^\beta} = \frac{1}{n^{\beta - 1}}
\]  

[8]

The interesting thing about the solutions given by eqn [8] is that the term \( 1/n^{\beta - 1} \) is the only difference between the numerator and denominator. This means that the marginal effect of irrigation mosaics can be determined from the value of \( \beta \). When \( \beta = 1 \) then \( I_{re} = 1 \) and the impact will be the same for irrigation mosaics and one contiguous area. However, when \( \beta > 1 \) irrigation mosaics will have a reduced impact and be advantageous, while for \( \beta < 1 \) irrigation mosaics will have an enhanced impact and be disadvantageous, compared to a contiguous irrigated area. This has now provided us with a powerful tool for analysing the various impacts that maybe occur if irrigation mosaics are introduced.

Here we will examine a number of bio-physical properties where we have determined the effect of the size of the patches on \( I_{re} \).
4.2. Water

Water is the essential driver in the hydrological system and in plant production. In terms of the water that humans consume, use and trade, the water imbedded in crops (sometimes called virtual water) is the largest amount by a large margin (Pearce, 2006). When we use water within agricultural enterprises, efficiency, timely and judicious use of this water will be required.

Water for irrigated agriculture has led to many problems in many parts of the world with; downstream users losing access, pollution of water moving downstream, water logging, salinisation, loss of social and economic equity and ground water rising or falling. In order to avoid these problems we need to plan irrigated agricultural developments based on well developed knowledge and facts.

4.3. Solutes

When soils are irrigated with water, that water will always contain solutes and the drainage water from the irrigated area will contain addition solutes from salts in the soil, fertilizers and other agri-chemicals. The addition of agri-chemicals and solutes in the irrigation water result in an additional volume of irrigated water being required to leach these solutes out of the soil (Cook et al., 2007). This means that there must be some additional discharge of water either to the ground water or to surface water if the area is not to be salinised. If the extra solutes are not removed from the irrigated area over time they will build up.

The removal also means that these extra solutes must be ‘disposed’ of safely somewhere else. These solutes may be discharged to the ground water where their addition may reduce the quality of the ground water. The may be drained by surface or subsurface drains and discharged to surface water, where they may reduce the quality of surface water.

No matter what the receiving environment for these extra solutes in designing an irrigation scheme their fate and cost will need to be accounted for. If this is not done then usually the cost is born by others either downstream or at a latter date.

4.3.1. Conservative, non-reactive solutes

Conservative solutes are those that do not react during their transmission through the soil/vadose zone/ground water or surface water. These can then be easily modelled if the water flow is known as they can be coupled linearly to the water flow. Examples of such solutes are chloride, tritium and bromine that are often used as passive tracers in solute transport experiments. The transport of solutes in porous materials can be described by the convective dispersion equation:

\[
\frac{\partial (\theta C)}{\partial t} = D_s \nabla^2 C - \nabla q C
\]  

[9]

where \( \theta \) is the volumetric water content of the soil [L³ L⁻³], \( C \) is the concentration of the solute averaged over a representative volume of soil [M L⁻³], \( t \) is time [T], \( D_s \) is the dispersion coefficient which combines molecular diffusion and hydrodynamic dispersion, \( \nabla \) is the div mathematical operator, and \( q \) is the Darcian flux of water [L T⁻¹]. When the boundary and initial conditions for eqn [9] are applied, eqn [9] can be solved by analytical or numerical methods.
There are many papers in the literature referring to solute transport in soils and some are summarised by Cook et al., 2007). In ground water aquifers the porous material is fully saturated and the $\theta$ term is no longer required. Equation [2] above can be rewritten to represent simplified steady-state advective flow of solutes from the base of the root zone to ground water by:

$$C(z) = C_R, \quad z \leq \frac{t_D}{R_\theta} + z_{RT}$$
$$= C_0, \quad z > \frac{t_D}{R_\theta} + z_{RT}$$

where $C_R$ is the concentration at the bottom of the root zone, and $C_0$ is the initial concentration at $t = 0$.

This kind of approach has been used with more sophistication by (Raats, 1974, 1975 and 1981) and offers a sound approach to salt transport in and below the root zone.

### 4.3.2. Reactive solutes

For solutes that react during the process of transport, the ability to model or simulate the transport becomes more difficult. For a linear decay process, the decay of the solute is given by:

$$C(t) = C_R \exp(-\alpha_c t)$$

where $\alpha_c$ is the decay constant; this results in a simple linear coupling of the decay process onto the transport process. For example, the modification to eqn [11] would result in:

$$C(z,t) = C_R \exp(-\alpha_c t), \quad z \leq \frac{t_D}{R_\theta} + z_{RT}, t < t_s$$
$$= C_0, \quad z > \frac{t_D}{R_\theta} + z_{RT}, t \geq t_s$$

For non-linear decay and retardation processes where the reaction rates are related to the concentration and soil properties the transport is more difficult to compute.

### 4.4. Lateral Flows in Aquifers

The transport of water and solutes in homogeneous aquifers is a well studied subject with many papers and books (Bear, 1972) on the topic. This transport has often been modeled by the use of the Bossinessq equation, which is often linearized by Dupuit-Forchheimer assumptions. This results in satisfactory results for the water table or free surface position but not the velocities. Knight (2005) has recently developed an extension of the Dupuit-
Forchheimer assumptions which allow not only good approximations of the free surface but also the velocities. Knight and Kluitenberg (2005) have shown how uncertainty can be included in the results for ground water flow for wells using Fréchet kernels. They also suggest that their results can be extended to multiple injection or pumping well configurations. The use of this analysis in any pumping or slug tests to determine aquifer properties is strongly recommended and the extension of this work to multiple well configurations would seem to be of importance if ground water extraction is to considered as the source of water in any irrigation projects.

Solute transport has usually been attempted by coupling the solute transport linearly to the water flow. Examples of such analytical solutions are those of (Raats, 1978a,b, 1983) where transfer function solutions using advection are presented. Dillon (1989) developed the DIVAST model which also allows linear decay of the solutes and dispersion to be accounted for by piecewise coupling of these processes. This solution would allow the calculation of a plume from an irrigation mosaic patch to be calculated and by using the principle of superpositioning the plumes from an array of mosaics to be calculated (see below of arrays). The problem with DIVAST is that it assumes that the irrigation patch does not perturb the ground water height. This is okay where a contaminant is applied at the soil but the hydrologic regime remains the same which is not the situation when irrigation is introduced. We will discuss below the fact that when irrigation occurs, the water table below the irrigated patch rises, and means that the velocity of the ground water in the immediate vicinity of the irrigated patch is no longer constant. DIVAST and other similar models maybe of use at some distance away from the patch where the velocity is approximately constant.

Recently Knight et al. (2002) and Rassam et al. (2004, 2005) have used a unit response (UR) approach to estimate salinity impacts on the Murray River. Rassam et al. (2005) considered the effects of uncertainty and assumptions in the UR approach and have shown that it is generally applicable and ease of use compared with MODFLOW simulations. This UR approach has been developed into a GIS based model (SIMRAT) which would be applicable to further investigation of irrigation mosaics. This analytical model will allow rapid evaluation of possible scenarios.
5. Analytical models

A number of analytical models are available to determine the ground water response to increased deep percolation due to irrigation on either circular or rectangular areas (Hantush, 1967; Dagan, 1967; Zlotnik and Ledder, 1992, 1993). These models are all developed from Hankel transforms and as such result in integrals that require evaluation. Zlotnik and Ledder, (1992) noted that the convergence of these integrals is slow due to the product of Bessels functions that they contain. We have developed a solution that used a Laplace transform and has resulted in an solution that is computationally much more easily evaluated.

5.1. Definition & assumptions

Hantush (1967) provided some solutions for steady recharge (vertical percolation) of water from rectangular and circular recharge areas (Fig. 4). These obviously can represent irrigation patches in a landscape. He derived solutions for both of these configurations and for both water table rise when recharge is occurring and water table decline when extraction is occurring.

Figure 4. Diagrammatic representation of ground water mound beneath rectangular and circular recharge areas.
Here we will only consider the water table rise for circular recharge zones. These circular patterns are illustrative of what rectangular patterns with aspect ratios close to 1 would also give. The solutions that Hantush derived (Appendix 1) are difficult to solve because they require integration of mathematical functions which are periodic and converge slowly. This was noted by Zlotnik and Ledder (1992) who developed criteria for judging when convergence occurs. There is a simpler approach to solving the problem posed by Hantush, using Laplace transforms and is given in Carslaw and Jaeger (1959) for a related temperature problem and this will be developed below.

5.1.1. Dimensionless parameters

The solution of the related heat problem has been developed by Lin (1967) based on Carslaw and Jaeger (1959). A solution was found using Laplace transforms and results in the following solution written in non-dimensional terms:

\[
H(\rho, \tau) = f(\rho, \tau) = \left( h - h_0 \right) \bar{b} K / \left( R^2 D_t \right)
\]

where \( H(\rho, \tau) \) is the non-dimensional water table rise, \( \rho = r/R \) is the non-dimensional radius, \( \tau = \bar{b} K / (S_y R^2) \) is the non-dimensional time, \( f(\rho, \tau) \) is a function (see below and appendix I), \( h \) is the dimensional water table height (Fig. 4) \([\text{L}]\), \( h_0 \) is the initial dimensional water table height (Fig. 4) \([\text{L}]\), \( R \) is the radius of the irrigated patch \([\text{L}]\), \( D_t \) is the recharge rate \([\text{L} \ \text{T}^{-1}]\), \( S_y \) is specific yield \([\text{L}^3 \ \text{L}^{-3}]\), \( \bar{b} \) is a linearisation parameter \([\text{L}]\) and \( K \) is the saturated hydraulic conductivity \([\text{L} \ \text{T}^{-1}]\). Equation [17] allows us to compute the problem in terms of non-dimensional space and time parameters. The function \( f(\rho, \tau) \) has the following set of solutions. For \( \rho = 0, 1 \) the solutions are exact and given by:

\[
f(0, \tau) = \tau \left[ 1 - \exp \left( -\frac{1}{4\tau} \right) \right] + \frac{1}{4} E_1 \left( \frac{1}{4\tau} \right)
\]

\[
f(1, \tau) = \frac{1}{2} \left[ \tau - \int_{0}^{\tau} I_0 \left( \frac{1}{2\alpha} \right) \exp \left( -\frac{1}{2\alpha} \right) d\alpha \right]
\]

where \( I_0 \) is a Bessel function of first kind and zero order and \( \alpha \) is a dummy variable. These solutions correspond to the centre of the irrigated patch (\( \rho = 0 \)) and at the edge of the patch (\( \rho = 1 \)). Approximate solutions were derived for the Laplace transforms and are given in appendix 1.

These functions are reasonably easily evaluated; a program was written in MatLab to determine \( H \). The next step is to revert to the dimensional parameters. Firstly we calculate the maximum height at \( \rho = r = 0 \) and then \( b \). From eqn [17] \( H(0, \tau) = H_m \) can be determined and from rearranging eqn [17] the value of \( h_m \) and \( b \) are given by:

\[
h_m = \sqrt{\frac{2H_m R^2 D_t}{K} + h_0^2}
\]

\[
\bar{b} = (h_m + h_0) / 2
\]

The physical (dimensional) time for these values is obtained from \( \tau \) by:
\[ t = \tau \frac{R^2 S_y}{bK} \] \[ [16] \]

Depending on the value of \( \tau \) either eqn \([A7]\), or \([A9]\) is used to obtain \( f(\rho, \tau) \) for the desired values of \( \rho \) and the values of \( r \) and \( h(r,t) \) obtained by:

\[ r = \rho R \]
\[ h = h_0 + \frac{HR^2 D}{bK} \] \[ [17] \]

This analysis allows the results to be presented in non-dimensional and dimensional formats to illustrate the impact of irrigation on the ground water. Firstly we will consider the rise of the water table in the centre of the patch in non-dimensional and then dimensional forms. Non-dimensional water table rise \( (H) \) as a function of \( \rho \) for various values of \( \tau \) show that at small values of \( \tau \) the value of \( H \) for \( \rho < 1 \) is equal to \( \tau \) (Fig. 5a). This is because the effect of the edge has initially a minimal impact on the rise which is confined to the area of the patch. As time increases the total volume of water added increases and the water table rise occurs at considerable distances away from the patch (Fig. 5b,c,d).

![Figure 5](image-url)  
*Figure 5. \( H \) as a function of \( \rho \) for various values of \( \tau \). Note that for small times (a) that the rises is vertical as the systems does not know about the edge yet, while at large times (d) the rise occurs to considerable distance from the source. Note the different x axis ranges used on all the sub-figures.*

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To examine the effects in dimensional terms we will take the values obtained for \( \tau = 1 \) and values of \( R \) varying from 100 m to 100 km. From eqn [16] we know that \( t \) increases with the square of \( R \) so that the solution for \( H \) with \( \tau = 1 \) is applicable at increasing time \( t \) as \( R \) increases. The results in Figure 5 represent an infinite set of \( h(R,t) \), time and \( R \) values contained in a single value of \( H(\rho, \tau) \).

The water table height as a function of \( \rho \) shows that as \( R \) increases the maximum height of the water table increase and the spread of the water table outside of the irrigated area \( \rho > 1 \) (Fig. 6) increases. These results show that the ground water rise for \( \rho > 4 \) is negligible. Rassam et al. (2004) in a report on salinity impacts of irrigation development found that the unit response equation was applicable when the no flow boundary was \( > 4 \) the width of the recharge (irrigation) zone. This is a similar result to that obtained here and suggests the no impact at a non-dimensional length scale of 4 may be a universal for such groundwater problems.

The times at which these water table profiles occur is however not variable. The maximum water table \( (h_m = h(r = 0)) \) at various values of \( R \) with time shows that as \( t \) increases \( h_m \) initially increases linearly (Fig. 7) and then increases progressively more slowly. What is interesting is that as \( R \) increases, \( h_m \) increases and the period of linear increase in \( h_m \) occurs for longer. For a 100 km patch \( h_m \) is still increasing linearly after 100 years and was 375 m above the initial level. These results are for a horizontal aquifer but Dagan (1967) showed that for sloping aquifers the result is essentially the same with only a slight displacement of the position of \( h_m \) down-slope during the early time. These results suggest that drainage

![Figure 6](image-url)
management and design should be integral to any large scale irrigation project, and monitoring of the water table height in the centre of any irrigation patch should be part of the management requirements. These results also indicate that when monitoring of water tables show a linear increase with time, the water table rise still is not approaching approximately steady-state and management options to cope with this continued increase should be considered.

Figure 7. Relationship between \( t \) and the change in the water table at the centre point of the irrigated area for various values of \( R \) varying from 100 m to 100 km. The other variables need to convert from non-dimensional to dimensional values are \( D_r = 1 \times 10^{-3} \text{ m day}^{-1} \), \( K = 1 \text{ m day}^{-1} \), and \( S_y = 0.1 \).

We can determine the scaling function for the rise in the maximum water table height from the data in Figure 7 using eqn [8]. This results in a very good fit between eqn [8] and the water table rise data (Fig. 8) and results in a value of \( \beta = 0.35 \). \( \beta < 1 \) which suggests that for patches, which are sufficiently isolated so that there is no appreciable interaction of the groundwater mounds, irrigation mosaics are disadvantageous. However, the scaling model is not appropriate for this problem as the property is not area related. What the results show is that as radius increases the water table height increases and the area affected by water logging is expected to increase. In order to analyse for this the specific depth at which water logging would occur needs to be known and then the area affected as a function of radius could be defined.
Figure 8. Maximum change in water table height \((h_m - h_0)\), \((h_m = h(r = 0))\) with radius \((R)\), with \(t = 100\) years and recharge rate and aquifer properties given in Fig. 7. The line is a regression of eqn [2] against the data, which results in a value of \(\beta = 0.35\), (regression coefficient of 0.999).

In order to define \(\beta\) to use the scaling process developed above we need to find the extent of the water table spreading beyond \(R\), as a function of \(R\). These were calculated so that the extent of the spread of the water table was taken as being when \(r(h - h_0) \approx 0.01h_0\). The results shown that the power relationship assumed by eqn [7] described the relationship between \(R\) and \(f - R\) with regression coefficients of > 0.997 (Fig. 9). The reason for plotting \(f - R\) as the independent variable is that \(C = R\) for the spreading of the water table mound.
Figure 9. Relationship between $\Delta r$ and $R$ for various values of $\tau$. The regression lines gave values of $\beta$ 0.58, 0.57 and 0.57 for $\tau = 0.75$, 5.0 and 10.0 respectively. The same values used in Figure 7 are used for other parameters.

The value of $\beta \approx 0.57$ indicates that the area impacted by water table spread is likely to be slightly more with irrigation mosaics than with one contiguous irrigation area. Although not shown here calculations were done to check that variation in the other parameters $I$, $S_y$, and $K$ did not affect this result. These results also need to consider the water table height when considering whether salinisation of areas surrounding an irrigated area is likely to occur. Given that small irrigated patches will raise the water table less, then there is likely to be less salinised soil with small patches and hence with mosaics.

5.1.2. Mosaics and Superpositioning

The effect of recharge on the water table when the water table rise from the patches that overlaps can be calculated. Due to the linearity of water table affects, the ground water response can be calculated in by treating each patch in isolation and then summing the rise due to each isolated patch to give the actual water table rise at a point of interest. A simple example is if we placed the recharge areas on top of one another the result would be the same as if we doubled the recharge rate. This ability to sum up the effect in linear systems is called the principle of superpositioning. We will use this principle to develop the solution for water tables when there is a regular pattern of patches in a homogeneous landscape.
This can be extended to non-uniform landscapes, where each patches could have different properties but the properties are homogeneous for a single patch this was not solved or presented here due to lack of time. This is obviously an extension of this work that should be considered when practical applications are required.

**Square Grid**

![Square Grid Diagram](image)

**Calculations on this line for Superpositioning**

Figure 10. Square grid used to calculate effect of mosaics. Due to symmetry calculations of water table height only need to be calculated along the line indicated. Grids extend to infinity away from the centre point.

We can thus use this principle to obtain the ground water patterns for any arrangement of mosaics using eqns [13-17]. However, it may be more useful to investigate the effect of some regular grid. For a square grid of points (Fig. 10) separated by \( L = nR \) where \( n > 1 \) then the value of \( H_n(\rho, \tau) \) is given by:

\[
H_n(\rho, \tau) = H_{n-1}(\rho, \tau) + H((nL-\rho), \tau) + H((nL+\rho), \tau) \\
+ 2 \sum_{i=1}^{n} \left[ H\left(\sqrt{(nL-\rho)^2 + (iL)^2}, \tau\right) + H\left(\sqrt{(nL+\rho)^2 + (iL)^2}, \tau\right) \\
+ H\left(\sqrt{(nL)^2 + ((n-i)L-\rho)^2}, \tau\right) + H\left(\sqrt{(nL)^2 + ((n-i)L+\rho)^2}, \tau\right) \\
- H\left(\sqrt{(nL)^2 + \rho^2}, \tau\right) \right]
\]

\[
H_0(\rho, \tau) = H(\rho, \tau) \\
n \in I
\]

[18]
where \( n \) is the summation level needed to calculate the water table height and is such that \( H(\rho = nL) \approx 0 \). The number of points involved in the determination of \( H_n(\rho) \) is \((2n+1)^2\). Equation (22) is applicable to where the properties of each irrigated patch are the same. It can easily be adapted to patterns where the properties but then the results will have to be calculated separately for each point in the grid. Another grid pattern where points are in a regular centred hexagonal pattern (Fig. 11) the value of \( H_n(\rho) \) is given by:

\[
H_n(\rho, \tau) = H_{n-1}(\rho, \tau) + H((nL - \rho), \tau) + H((nL + \rho), \tau) \\
+ 2 \sum_{i=1}^{n} \left[ H\left(\sqrt{([2n-i]x - \rho)^2 + (iy)^2}, \tau\right) + H\left(\sqrt{([2n-i]x + \rho)^2 + (iy)^2}, \tau\right) \right] \\
+ 2 \sum_{j=1}^{m} H\left(\sqrt{(ny)^2 + [n-2i]x - \rho}^2, +, \tau\right) + H\left(\sqrt{(ny)^2 + [n-2i]x + \rho}^2 +, \tau\right) \\
- jH\left(\sqrt{(ny)^2 + \rho^2}, \tau\right)
\]

\[19\]

\( x = L/2, \quad y = L\sqrt{3}/2 \)
\( n = 2, 4, 6, ..., \quad m = n/2, \quad j = 1 \)
\( n = 1, 3, 5, ..., \quad m = (n-1)/2, \quad j = 0 \)

The non-dimensional \( H \) can then be converted to \( h(r,t) \) using eqns [13-17].

**Centred Hexagonal**

Calculations on this line for Superpositioning

**Figure 11.** Centred hexagonal grid used to calculate effect of mosaics. Due to symmetry calculations of water table height only need to be calculated along the line indicated. Grids extend to infinity away from the centre point.

A computer program was written in MatLab to solve eqn [18] and the effects of the spacing between the irrigated patches (\( L \)), the size of the patches (\( \rho \)) and the time since irrigation had started (\( \tau \)) on the water table height (\( H \)) examined.
The results are presented in non-dimensional variables, so it is worthwhile discussing what these mean. When a small value of non-dimensional time ($\tau$) is discussed below this means small for that particular system. In dimensional time ($t$) when $\tau$ is considered to be small the real time will vary with $R^2$ and can be large for large $R$ (Table 1). $t$ is linearly related to $K$, $S_y$, and $b$ (eqn [16]).

Table 1. Comparison of non-dimensional and real time for various values of $R$. The values of other variables used in calculating the results in this table are: $h_0 = 10$ m, $K = 1$ m day$^{-1}$, $I = 1$ mm day$^{-1}$ and $S_y = 0.1$ m$^3$ m$^{-3}$.

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>$t$ (day)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>R = 100 m</td>
</tr>
<tr>
<td>0.1</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>10</td>
<td>995</td>
</tr>
<tr>
<td>100</td>
<td>9921</td>
</tr>
</tbody>
</table>

What we can see is that for an irrigated patch with $R = 100000$ m (100 km), a short non-dimensional time of $\tau = 0.1$ relates to a very long real time of 9995104 days (27384 years). This difference in perspective from the system to real time needs to be kept in mind when interpreting these results.

The first point to notice is that as $L$ increases the value of $H$ tends to that of an isolated patch ($L = \infty$) (Figs 12, 13). However, as $\tau$ increases the value of $L$ at which $H$ tends towards that of the isolated patch increase. This information could be used in designing and creating rules (policy) for regions about how many and how closely irrigated patches could be spaced. It must be remembered though that the mathematics used here is for an infinite array of patches and is likely to overestimate the effect of overlapping of water tables at long times.

When $L = 2R$ the patches just touch and $H$ tends towards that which would be found under a contiguous irrigated area with only minor ripples when $\tau$ is small (Fig. 12). What this shows is that at large times the water table is essentially flat (Fig. 13), which is what would be expected.
Figure 12. \( H \) versus \( \rho \) for a) \( \tau = 0.1 \) and b) \( \tau = 1 \).
Figure 13. $H$ versus $\rho$ for a) $\tau = 10$ and b) $\tau = 100$. The scales for both axes are in log format to allow the range of values to be seen.

The value of $H$ at $\rho = 0$ is inversely related to the value of $L$ and directly related to $\tau$ (Fig. 14). What this strongly indicates is that spacing of irrigation patches will have a very marked effect on the height that the water table will rise to. This means that the number, spacing
and size of irrigation patches will need to be carefully planned if adverse water table effects are not to occur in a region. It must be noted though that these results do not consider extraction from the water table. The analysis presented here could be extended to include a mosaic of extraction and injection (irrigation patches) in the landscape.

![Graph](image)

**Figure 14.** The increase in the water table height at the centre of the patch for a square grid mosaic compared to that for an isolated patch \( \frac{H(\rho = 0)}{H_0} \) versus the mosaic spacing \( L \), for various values of \( r \).

Time has not permitted the development of a computer program to solve the hexagonal mosaic pattern, but the results will be essentially the same as for the square grid.

### 5.2. Solutes

To determine the transport of solutes would require the development of particle tracking programs associated with the velocity of water flow. These can be developed using the solutions developed above along with those of Zlotnik and Ledder (1992,1993). A simpler method was developed here, which should at least provide the bounds of where solutes are likely to be advected to. These assume that the water reaching the water table either displaces completely all of the resident water that was there prior to the change in the drainage rate (Fig. 15a) or sits on top of the water that was initially present (Fig.15b).
Figure 15. Diagram of the two advective solute transport schemes used to estimate the position of the solute front: a) complete displacement of old water by new water to the impermeable base and b) the new water sits on top of the old water.

The depth of water ($V_i$) added is simply given by:

$$V_i = D_i \Delta t$$  \hspace{1cm} [20]

The depth of water stored in the water table ($V_s$) for the two cases above as a function of radius is given by:

$$V_s(r, t) = \int_0^r \Delta h(r) \theta_s dr$$  \hspace{1cm} [21]

where $\Delta h(r) = h(r) - h_0$ for the case given by Figure 15a, $h_0$ is $h$ at time $t$, $\theta_s$ is the saturated volumetric water content of the aquifer and $\Delta h(r) = h(r) - h_0$ for the case given by Figure 15b. The radius the solute has advected to is determined by finding the value of $R_s$ at which $V_s = V_i$. These calculations need to be performed in dimensional variables, so the examples below only cover a limited range of scenarios.
Figure 16. The effect of radius (R) on the proportional radial distance that solutes have advected to (Rs/R) for a) scenario 1 defined in Figure 15a, and b) for scenario 2 defined in Figure 15b for various times and values of water content change in the aquifer (Δθ).

The results indicate that with $h_0 = 10$ m and using scenario 1 (advection to bottom of aquifer) the ratio of the distance the solutes have advected beyond the radius of the patch...
(R) decreases with increasing R (Fig. 16a) and this decreases is more pronounced as time increases (the slope is steeper). For scenario 2 (advection on top of existing watertable) a decrease in the \( R_s/R \) also occurs with increasing \( R \), but the slope of this decrease is steeper (Fig. 16b).

The results presented are for changes in water content of the aquifer (\( \Delta \theta \)) of 0.1 and 0.3 m³ m⁻³. For scenario 1 with \( \Delta \theta \) and \( t = 100 \) years, \( \beta = 0.31 \) which implies that the irrigation mosaics will result in less area being impacted than would occur with one large patch with regard to spread of solutes into the surrounding area. For the second scenario with \( \Delta \theta \) and \( t = 100 \) years, \( \beta = 0.14 \), which indicates that irrigation mosaics will also result in more area being impacted by the spread of solutes than one large patch (Fig. 17). This result is in the same direction as the results for leakage rate from saline basins (Paydar et al. 2007; Leaney et al. 2000) of \( \beta = -0.5 \), but not as strongly negative for irrigated mosaics. However, in the case of saline basins the water level is maintained in the basins and then the out flow rate will decrease as the perimeter to area ratio changes with radius and a value of -0.5 is expected for \( \beta \) (Wooding, 1968). The fact that the water table is likely to be lower for smaller patches tempers the interpretation of this result as a negative result for irrigation mosaics.

![Figure 17](image.png)

**Figure 17.** \( f - R \) with R for scenario 1 (S1) and scenario 2 (S2), for various values of \( R \) with \( \Delta \theta = 0.1 \). The lines are linear regressions and the slope of these corresponds to \( \beta \).

Another factor that will affect scenario 1 is the value of \( h_0 \), obviously as \( h_0 \) tends towards zero then \( R_s/R \) values for scenario 1 will tend towards those of scenario 2 and as \( h_0 \) increase towards infinity then \( R_s/R \) will tend decrease (Fig. 18). Also as \( h_0 \) increases this effects \( R_s/R \) for scenario 2, but now \( R_s/R \) increases.
The results here provide bounds to the possible solute behaviour that may occur and provide estimates of the scaling parameter $\beta$. For reactive solutes where the behaviour can be described as linear with time i.e.

$$C(t) = C_0 \exp(-kt)$$

where $C(t)$ is the concentration at $t$, $C_0$ is the concentration at $t = 0$ and $k$ is the reaction rate parameter, these results will apply with just a shift along the time axis. More complicated results will occur when non-linear processes, where the reaction rate is govern by the concentration. In Kartsic aquifers where zones of high hydraulic conductivity and/or tunnels can occur further non-linearities will occur that cannot be accounted for in the above analysis. These tunnels could act as fast conduits for water and solutes or could cause the water to flow around them depending on their size, shape and the hydraulic conductivity of the surrounding material (Philip, 1989). Irrigation mosaics may provide the possibility to avoid irrigation on such areas and possible rapid transport of solutes.

### 5.3. Transient systems

The results given above are for a steady percolation rate from the irrigated patch but the analysis could be extended to allow for period changes in the percolation rate. Singh (2006)
has provided a solution for sinusoidal variation in the pumping rate from wells and this can be adapted to periodic recharge. This extension would require some time to develop but could be a very useful result in highly periodic systems like the northern wet/dry climatic sequence. The extent to which this periodicity will be smoothed out by irrigation requires investigation. The numerical results presented in section 4 below does take into account the periodicity of the recharge system and these results are indicative of what will periodic recharge will produce.

Since groundwater would often be the source for irrigation in northern Australia the analysis of Singh (2006) on groundwater levels and that of Singh (2005) on depletion of stream flow would also be useful when an actual systems is being considered for development. Again for an array of wells the superpositioning principle can be used to determine the overall effect of multiple wells.

5.4. Drainage: Surface and deep drainage

5.4.1. Surface Drainage

Numerous analytical models exist for drainage of soils. These are generally based on solution of the Boussineqs equation. Recently Singh et al. (in prep) have developed some solutions which allow for evaporation and recharge at the same time in rectangular blocks with drainage on all sides.

During the wet season the drainage is likely to be accompanied by high evaporation rates and some of the tradition drainage equations developed for Europe conditions will not be suitable (Cook and Rassam, 2002). The solutions of Cook and Rassam (2002) are suitable for the drainage to parallel drains and as these are presented in non-dimensional terms will be efficient in determining drain spacing for design purposes (Fig. 19). They also show that in climatic zones where the evaporation rate can be high the contribution of water table decline due to drains can be small except in soils with high saturated hydraulic conductivities.
5.4.2. Deep drainage

Deep drainage from the irrigated areas can be estimated or simulated with any number of numerical water balance models. Recently Cook and Knight (2007) have developed models to determine the monotonic drainage of soil profiles with time. These are based on the analytical models of Broadbridge and White (1987). They do not explicitly give the volume drained but this can be easily inferred from the water content profiles. For estimation of deep drainage it will be more sensible to use numerical water balance models and a number are listed in Cook et al. 2007.

Cook et al. (2007) also stated the need for drainage to prevent the salinisation of irrigated soils. Depending on the built up of salt during the dry season and the crop sensitivity to salt build up, wet season leaching may be enough to remove the salts. A combination of modelling with initially a model like Raats (1974), experimental observations and numerical modelling would be prudent in any initial investigations.
5.5. Key findings

1. The water table rise under an isolated circular irrigation patch increases with the radius of the patch exponentially. For irrigation mosaics as the patches are placed closer together the water table rise tends towards that of a single larger patch. Although an infinite array of patches was modelled here it is easy enough to truncate the summations for smaller arrays.

2. By using a judiciously spaced mosaic pattern the water table rise can be reduced compared to one large patch where the irrigated area is the same for the mosaic and the single patch. The models provided here offer some guidance in designing such mosaics. The scaling parameter for water table rise was found to be, $\beta = 0.35$

3. Dagan (1967) showed that the results presented here for a flat aquifer will be equally as valid for a sloping aquifer, except at early time where some downslope displacement of the water table height peak occurs.

4. The superpositioning arrays developed here could be used to look at combined irrigation patches and extraction wells to determine optimal designs for minimising environmental impacts.

5. The scaling parameter for comparison of isolated patches gives $\beta \approx 0.57$ which suggests a small advantage for a contiguous area in terms of the area affected by water table rise. This assumes that the mosaic spacing is such that no overlap occurs.

6. The area impacted by the spread of solutes outside of the irrigated area, within the ground water, is more for a mosaic of small patches with the same area as a single large patch than for the single large patch. However, this result needs to be tempered by the possibly lower water table depth for the mosaics. The scaling parameter $\beta$ for solute spreading varied from 0.14 to 0.31.

7. The solutions for steady percolation could be extended to allow for periodicity in the recharge rate.

8. With correct spacing and design irrigation mosaics could reduce the water table rise, ground water spreading and solute spreading and may be a useful tool for more environmentally friendly irrigation schemes. However, if designed poorly they will behave in a similar way to a large contiguous irrigation system.
5.6. Conclusions and future needs in relation to analytical models

The results here show that the maximum water table rise due to increased percolation (deep drainage) to a water table will be significant under large irrigation areas or mosaics where the spacing is close together. These show that if a water table is rising in the middle of the irrigated area in a linear manner then the increase is still going to increase for some time to come. This could be a useful monitoring tool in irrigated areas.

Only limited solute transport was attempted but this showed that the area impacted by transport of solutes outside the irrigated patch could be increased by smaller patches, but this would need to be carefully evaluated, as water table rise would also have to be taken into account.

The results presented indicate that, using the scaling theory developed earlier, irrigation mosaics are likely to have an advantage with regard to the maximum height of water table rise, and potentially a slight disadvantage with regard to water table and solute spreading, compared to one contiguous irrigated patch. This statement assumes that the total area irrigated by the mosaics is the same as the contiguous patch, homogeneity of soil and aquifer properties occurs, and that the recharge rate is the same for both the mosaic and contiguous area. To determine the actual effect of careful selection of mosaic sites is beyond this analysis.

The water table rise results highlight the need to incorporate drainage as part of any irrigation design. Drainage is only given cursory attention here as it is covered in detail in many other publications. The water table rise results do however suggest that irrigation mosaics could be used to minimise the need for drainage requirement if the spacing and size of the irrigation patches were chosen judiciously.

We have presented new solutions for water table rise which will have utility in both this and many other applications. They offer the ability to develop some design tools that will compliment existing numerical modelling programs such as MODFLOW. The further modelling need is:

1. Development of a particle tracking program based on Zlotnik and Ledder (1991a,b) to give better understanding of solute movement.
2. Extension of the present solutions for water table rise to include periodic variation in recharge rate.
3. Development of a program to solve the superpositioning problem for the hexagonal grid.
4. Modelling using the grids of arrays of extraction wells and irrigation patches.
5. Superpositioning models for truncated array of patches and for patches with different properties.
6. Numerical models

Numerical modelling of ground water systems is an essential component in the decision making concerning water resources management. They are indispensable in the planning, implementation and adaptive control of decisions regarding recharge, extraction, siting of wells and the general operation of the water resource system. Most numerical ground water models in use today are of the predictive type while others are more suited to testing and evaluating hypothesis. Most ground water models are designed to solve partial differential equations of ground water flow and the advection-dispersion equation. The adoption and use of these models depend on the results of user evaluation of the model’s conceptual and mathematical framework, the computer software and documentation and availability. MODFLOW is a modular three dimensional finite-difference ground water flow model of the U. S. Geological Survey to describe and predict the behaviour of ground water flow systems. The solute transport module MT3DMS is a transport model that uses a mixed Eulerian-Lagrangian approach to the solution of the three-dimensional advective-dispersive-reactive transport equation. MT3DMS is based on the assumption that changes in the concentration field will not affect the flow field significantly. This allows the user to construct and calibrate a flow model independently. After a flow simulation is complete, MT3DMS simulates solute transport by using the calculated hydraulic heads and various flow terms saved by MODFLOW. MT3DMS can be used to simulate changes in concentration of single species miscible contaminants in ground water considering advection, dispersion and some simple chemical reactions.

6.1. Heterogeneity

6.1.1. Space
Aquifer systems are inherently heterogeneous in space with respect to its properties such as saturated hydraulic conductivity, specific yield, specific storage etc. Aquifer lithology and hydraulic conditions are known to vary in space to a large extent. In fact each of the parameters that describe the media can vary through space. The hydraulic conductivity and aquifer dispersivity may have directional properties and need to be specified throughout the region of interest. The definition of the geometry and the distribution of the parameters in space as well as the domain boundary conditions depend on a thorough understanding of the geology of the area. Modelling of real system is done by simplifying and abstracting these systems into parameter distributions and boundary conditions which make the problem tractable mathematically.

6.1.2. Time
Aquifer properties like density of water may vary with time but most numerical models ignore this variation as the produce addition layer of complication and data requirement.

6.2. Limited investigation of monolithic versus mosaic irrigation systems
The concept of mosaic irrigation systems as opposed to single monolithic systems can be investigated using a number of spatial numerical models some of which are linked to a GIS framework. In this work we used a simple ground water model (MODFLOW) to investigate
the effect of mosaics on the ground water system as compared to a single monolithic system. The comparison will be based on a hypothetical homogenous unconfined aquifer with the following characteristics:

The model area is 120 X 120 km with one layer 20 m thick and bounded on all four sides by a free flow boundary. The free flow boundary condition means that flow out of the area occurs if the watertable rises above the initial value in the cells along the boundary. For the purpose of this analysis the area may be assumed to be of an infinite extent. The aquifer is unconfined and is assumed to be coarse grained sand with a hydraulic conductivity of 160 m/d and specific yield is 0.06. The aquifer is excited with three cycles of recharge and no recharge. The recharge period was 120 days and the zero recharge period was 240 days. The recharge periods were configured into six stress periods of alternating wetting and drying. Hence transient flow simulation was conducted for a time period of three years. This scenario is assumed to mimic irrigation events. The patch configurations are shown in the below (Fig. 20). The initial hydraulic head for all simulations was fixed at 5 m above an arbitrary datum for the entire domain.

![Figure 20. Hexagonal (A), rectangular (B), and single patch configurations.](image)

The spacing between the rectangular mosaic patches is 17.5 km \((L \approx 7R, R = 2500 \text{ m})\) and for the hexagonal mosaic patches is 21 km \((L \approx 7R, R = 3000 \text{ m})\)
6.2.1 Ground water levels and Solute concentrations

A recharge rate of 0.002 m/d was applied to each of the patches in all the configurations and ground water levels were simulated. Advection-dispersion transport was simulated assuming that the recharge in the patches have a conservative solute with a concentration of 4320 µg/m³. The longitudinal dispersivity was 30 m and the ratio of horizontal transverse dispersivity to longitudinal dispersivity was 0.3. Similarly the ratio of vertical transverse dispersivity to longitudinal dispersivity was 0.3. Complete mixing into the ground water system is assumed.

6.2.1.1 Single area configuration

Irrigation causes a rise to occur in the ground water under the patch and with increasing distance away from the patch with increased irrigation (Fig. 21). These results show similar behaviour to those found with the analytical model but we have also been able to incorporate fluctuations in the recharge rate.

These fluctuations in recharge rate result in the ground water height showing a saw-toothed behaviour with regard to water table rise with time (Fig. 22). This is due to the periodicity in the recharge rate used in these simulations. The amplitude of the fluctuation in water table height decreases with increased distance from the edge of the irrigated patch. The height of the water table rise decreases as the distance from the centre of the patch increases. There is also a shift in the time when the peak occurs which increases with distance from the edge of the irrigated patch.
Figure 21. Ground water heights at the end of a) stress period 1, b) stress period 3 and c) stress period 6.
Figure 22. Ground water height variation with time at the centre of the single patch (r = 0), 500 m from the edge of the recharge area (r = 8000 m) and 2000 m from the edge of the recharge area (r = 9500 m).

The solute concentration by comparison with the water table height shows a ramp up in concentration associated with each recharge period and then an approximately constant value (Fig. 23). The amplitude of the ramp decreases as the distance from the irrigated patch increases. The time before solutes are simulated to arrive at a radius increases as the radius increases (Fig. 23).
Figure 23. Solute breakthrough curves at 500 m from the edge of the recharge area for a single patch \((r = 8000 \text{ m})\) and 2000 m from the edge of the recharge area for a single patch \((r = 9500 \text{ m})\).

6.2.1.2 Rectangular mosaic configuration

The patches initially do not ‘know’ of each other in the simulations for the square grid and act as isolated patches (Fig. 24a) but by stress period 3 the mounds resulting from the rise in water tables have coalesced (Fig. 24b) and by stress period 6 are starting to have the appearance of one large irrigated patch (Fig. 24c).
The recharge rate for the patches in the square grid is the same as that to the single patch. The maximum ground water height at the centre of the grid under one of the mosaics patches does not rise as high as in the single patch after stress period 3 (Figs 22 and 25). At stress period 3 there is no overlap, so the results are the same as would be obtained for a
isolated patch with $R = 2000$ m. This supports the results for maximum ground water height in Figure 8. The water table height peaks now have curvilinear rise and fall shapes (Fig. 25), which is due to the smaller size irrigated patch. The spacing between the patches is approximate $7R$ so that little overlap will occur initially as is shown.

![Graph showing ground water level variation with time at different distances from the center patch.](image)

**Figure 25.** Ground water level variation with time at the middle of the center patch, at 500 m from the edge of the center patch and at 2000 m from the edge of the center patch for the square mosaic grid.

There is also a much longer time delay before the solute breaks through at 2000 m from the centre of the central patch (Fig. 26). The concentrations are also much lower at the same stress period at both distances for the mosaics compared to the single patch (Figs 24 and 26).
Figure 26. Solute breakthrough curves 500 m ($r = 3000$ m) and 2000 m ($r = 4500$ m) from the edge of the patch for the square mosaic grid.

6.2.1.3 Hexagonal mosaic configuration

The hexagonal grid results in the patches not overlapping in the first stress period but overlapping has occurred by stress period 3 (Fig. 27a, b). The water table height pattern is almost that of a circular single patch by stress period 6 (Fig. 27c).
Figure 27. Ground water heights (m) for a hexagonal grid of irrigation patches at the end of a) stress period 1, b) stress period 3 and c) stress period 6.
The height of the water table rise at the end of stress period 1 is between that for the single and square grid (Fig. 28). This is what would be expected, as the radius of the patch in the hexagonal grid at 3000 m lies between the 2500 m for the square grid and 7500 m for the single patch. The peaks are similarly curvilinear in shape on the rising and falling limbs as for the square mosaics. This confirms the earlier assertion in the analytical modelling that the results for the square grid will be similar for the hexagonal grid.

**Figure 28.** Ground water height variation with time at the middle of the center patch, at 500 m and 2000 m from the edge of the centre patch.

The solute breakthrough curves are again similar to those for the square grid but the concentrations are higher and break through times are earlier (Fig. 29).
Figure 29. Solute breakthrough curves 500 m and 2000 m from the edge of the patch for a hexagonal grid of irrigation patches.
6.3. Key findings

The key findings from the numerical modelling are similar to those from the analytical modelling except that the effect of periodicity can be ascertained. The findings already given for the analytical modelling will not be repeated here.

1. The periodic recharge results in reduction in the rate of rise of the water table with time, however this is only a temporary reprieve, as the time course is the same as if the recharge was constant.
2. The mosaics show that they can result in lower water tables and solute transport than a single irrigated patch. However, this is only for a single level of mosaics.
3. The mosaic results here for a single level is the other extreme to the infinite array in the analytical modelling and these two approaches provide bounds on the mosaic problem.

6.4. Conclusions and future needs in relation to numerical models

The results here show that mosaics could be useful in lowering the overall water table rise in a region which is consistent with the analytical results. The numerical analyses also demonstrated that solute spreading will be greater under mosaics but that the solute concentrations will be lower, results consistent with those found using the analytical solutions. A combination of numerical and analytical modelling will provide a means of determining how best to design actual irrigation mosaics. Future simulations should include:

1. Simulations incorporating heterogeneity in the aquifer properties
2. Mosaics with a greater number of patches
3. Reactive solutes, involving both linear and non-linear exchange processes
7. System and climate effects

When considering irrigation systems one needs to consider the climatic conditions associated with the region. Most Australian irrigation systems and hence most Australian irrigation experience is with the climate in Southern Australia. Northern Australia’s climate is quite different being monsoonal, which means there are distinct wet and dry periods. The wet season also coincides with summer and higher evaporation. Petheram and Bristow (2008a, b) have recently reported on the hydrology of northern Australia and for a more complete overview readers are referred there.

7.1. Surface irrigation (source of water: rivers and dams)

Extracting from surface river sources requires that there is ‘excess’ water to harvest from the river system. Petheram and Bristow (2008) suggest that there may be less water to harvest from river systems than was originally thought and that this needs further investigation. The seasonality of the climate in northern Australia means that when rivers are flowing irrigation will generally not be needed. This means that if the source of water is to be surface water, then dams will be required to store this water.

For dams to be built the geology, topography and climatic factors need to be taken into account. In much of northern Australia the relief is flat, so finding suitable storage sites could be difficult.

Evaporation rates in northern Australia can mean that large evaporative losses of water occur. It is estimated that 1870 GL of water in lake Argyle is lost to evaporation annually (Ruprecht and Rogers, 1999), which is 6 times that used for irrigation.

7.1.1. Delivery systems

Irrigation mosaics may offer the ability to use different delivery systems such as piped water which is not usually the case for large irrigation areas where channels are the usual means of delivery. This means that losses due to evaporation and seepage from channels could be reduced if mosaics were used. However, this may come at a price.

7.2. Ground water based irrigation (source of water: bores)

Ground water is a likely source for irrigation in northern Australia, as many rivers do not flow in the dry season (Petheram and Bristow, 2008). Ground water extraction will have to be carefully managed as this forms the base flow in rivers like the Daly and taking from ground water will have an effect on this base flow.

Over-extraction of ground water has already caused problems in the Burdekin Delta, with salt water intrusion occurring (Petheram et al., 2008). Such problems suggest that a well managed system of allocation will be required based on sound scientific and engineering knowledge. The danger in a distributed system such as irrigation mosaics is that the response may initially not be seen and time lags could occur. This could lead to detection of a problem at a point where economic resources have been sunk into an ecologically unsustainable development. The other danger is the ‘tyranny of small decision’ where additional ground water is approved without fully considering the total volume of extraction that is already occurring. Heckbert et al. (2006) explore different trading scenarios as a means of managing the irrigation from the Tindall aquifer near Katherine. This showed that unconstrained exploitation would not be possible.

The recent analysis of Singh (2005, 2006) offer methods for calculating the depletion on river flows of ground water extraction. A report on the topic of ground water/surface water
interaction is being prepared by Rassam et al. (2007) and will offer guidance in the future on methods for calculation of ground water extraction on surface water sources.

7.3. Seasonality

The climate in northern Australia is characterised by the seasonality resulting from the monsoonal influence. This results in a wet summer season and dry winter season which is unlike the climate associated with Southern Australia. This seasonality results in a highly periodic hydrology. The introduction of irrigation will result in some smoothing of this periodicity. Mosaics, depending on their number and location, could offer the possibility to minimise disturbance of this periodicity. The water table arrays shown above indicate how the mosaic spacing could affect the water table in the surrounding areas. The use of this model with a periodic solution would offer a way to determine alternative designs.

7.3.1. Rate of change (northern vs southern systems)

The climate data from three Southern locations (Loxton SA, Tatura VIC and Griffith NSW) and three Northern locations (Kununurra WA, Katherine NT and Burdekin QLD) have been used to illustrate these differences. The data was derived from both the Bureau of Meteorology site and the SILO database. The monthly rainfall and potential evapotranspiration ETo (Allen et al., 1998) were obtained over a 118 year period and the mean 10 and 90 percentiles calculated.

The rainfall is relatively evenly spread throughout the year for the Southern sites while the ETo shows a sinusoidal variation on an annual basis (Fig. 30). The monthly potential water balance was calculated as rainfall minus ETo for mean values, and the two extreme values; 10 percentile rainfall minus 90 percentile ETo and 90 percentile rainfall minus 10 percentile ETo. This shows that there is a deficit in the monthly water balance through most of the year. The water balance deficit is low in winter but high in summer and is due to the change in ETo.
Figure 30. Rainfall (top panel), ETo (middle panel) and water balance (bottom panel) for three Southern Australian sites viz Loxton, Griffith and Tatura. The water balance is the monthly potential value calculated by subtracting monthly ETo from rainfall.

For the northern Australia sites the patterns are different with most of the rainfall occurring in October to April and almost no rainfall in the remaining months of the year (Fig. 31). The amounts of monthly rainfall are also distinctly larger than for the Southern sites. The ETo is now more uniform throughout the year with only small sinusoidal variation. Also the peak in ETo now occurs in October rather than the December/January peak for the Southern sites. This results in monthly potential water balances with deficits for approximately 8-9 months of the year. Except for the lower extreme case positive water balances occur in the wet season, which would result in generation of a potential surface or ground water irrigation source.
Figure 31. Rainfall (top panel), ETo (middle panel) and water balance (bottom panel) for three northern Australian sites viz Kununurra, Katherine and Burdekin. The water balance is the monthly potential value calculated by subtracting monthly ETo from rainfall.

There is also much less variance in the Northern water balance compared to the Southern water balance. This is because there is less variance in the rainfall. The total annual water balances for the two systems show that the Southern sites always have a potential annual water balance deficit (Table 2), while this was true for Kununurra the other two Northern sites for the wettest scenario had small (Katherine) or large (Burdekin) possible potential water balance surpluses. This suggests that there could be large intra-annual variations in the need for irrigation. These values will be altered when the evapotranspiration is calculated by applying an appropriate crop factor. Depending on the crop and site this will modify the values in either increasing or decreasing the water balance values. Further analysis is required on this topic to better evaluate the potential or otherwise of irrigation in northern Australia.
Table 2: Annual potential water balances for three Southern and three northern Australia sites.

<table>
<thead>
<tr>
<th>Water Balance based on</th>
<th>Southern Sites</th>
<th>Northern Sites</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Loxton</td>
<td>Griffith</td>
</tr>
<tr>
<td>Means</td>
<td>-925.533</td>
<td>-981.846</td>
</tr>
<tr>
<td>90 Percentile rain – 10 percentile ETo</td>
<td>-495.25</td>
<td>-389.18</td>
</tr>
<tr>
<td>10 Percentile rain – 90 percentile ETo</td>
<td>-1284.68</td>
<td>-1487.79</td>
</tr>
</tbody>
</table>

7.4. Advection (patch size evaporative demand)

Enhanced evaporation of water due to advection caused by hot dry air moving over a wet patch or at the edge of irrigated areas is a well known phenomena (Priestley, 1955; McNaughton, 1983; Lang et al., 1983; Kadar and Yaglom, 1990). There is little research on this particular topic and disagreement between researchers as to its importance. Priestley (1955) and more recently Kadar and Yaglom (1990) suggested that the convective boundary layer is likely to remain disturbed and not reach equilibrium for a considerable distance into an area where there is an abrupt change in water vapour and or heat flux. Opinion is divided on this issue with some colleagues suggesting that a 10-20% increase in evaporation rate may occur. Another is that this is unlikely as entrainment of water vapour from patches within the region will get mixed by convection and there will be little enhancement in overall evaporation. Some guidance can be obtained from the review of evaporation from water bodies by Sweers (1976).

The evaporation rate \( E \), using the Dalton equation, from a water body is (Sweers, 1976):

\[
E = -f(u)(e_s - e_a)
\]

where \( e_s \) and \( e_a \) are the saturated and actual vapour pressure deficit respectively and \( f(u) \) is the wind speed function. This same Dalton equation for evaporation has also been successfully used by Conway and van Bavel (1967) to calculate the evaporation form a soil surface. Sweers (1976) reviewed the existing literature and concluded that the wind speed function was best given by:

\[
h(u) = \left( \frac{a_v}{S} \right)^{0.05} (b_v + c_v u) = S^{-0.05} A_v
\]

\[
A_v = a_v^{0.05} (b_v + c_v u)
\]

where \( a_v, b_v \) and \( c_v \) are empirical constants (see Sweers (1976) for values), \( S \) is the representative area of the evaporating area and \( u \) is windspeed. Assuming that the patches
are circular then \( S = \pi R^2 \) and that \( u \) is the same, then the effect of \( R \) on evaporation rate can be obtained from eqn [24]. The comparative evaporation rate can be calculated by:

\[
\varepsilon = \left( \frac{S_r}{S} \right)^{0.05} = \left( \frac{R_r^2}{R^2} \right)^{0.05}
\]

[25]

where \( S \), is a reference area and \( R \), is a reference radius if the areas are circular. Taking \( R_r = 100 \text{ km} \) results in a doubling in the wind factor when the radius reduces to 100 m (Fig. 32).

![Figure 32](image)

**Figure 32** \( f(u)/A_e \) versus \( R \) and \( \varepsilon \). \( R_r \) is taken as 100 km when calculating \( \varepsilon \). The arrows indicate which scale applies to which data. The line through the solid symbols is a regression of eqn [24] and gives \( \beta = -0.1 \) with a regression coefficient = 1.

This does not result in a doubling of the evaporation as there are feedbacks and other effects. The predicted evaporation rates are by comparison only increased by a factor of 1.28 or 28% (D. McJannet *pers. comm.*, 2007). This is still large and the effect of the size of irrigation patches on evaporation and hence water use should be experimentally investigated.

The scaling factor \( \beta = -0.1 \) and indicates that irrigation mosaics may result in higher evaporative losses and may have lower water use efficiency. The value of \( \beta \) is derived for isolated patches within an otherwise unirrigated landscape. In a mosaic situation there would be other patches within the landscape and the advective enhancement of evaporation would depend on mixing of the air mass at a regional scale. Thus for mosaics the enhanced evaporation due to advection is likely to be reduced compared with a single patch.
8. General Discussion and Conclusions

This report has developed some tools for analysing mosaics and provided direction for further research on irrigation mosaics. The scaling approach developed in section 2 can now be used to assist in further assessing the potential advantages and disadvantages of irrigation mosaics.

The scaling parameter $\beta$ for various processes that has been calculated here is given in Table 3 along with some aspects of mosaics that can easily be estimated. An advantage to neutral outcome occurs for four of the seven properties examined in this analysis of irrigation mosaics. This analysis has however only examined a few of the issues that need to be canvassed when considering mosaics. The modelling that was done here, the recent information in other reports (Heckbert et al. 2006; Paydar et al., 2007; Petheram and Bristow, 2008; Petheram et al. 2008; Rassam et al. 2007) when put together with other information on social and economic outcomes should allow improved understanding of what in reality is a very complex system (Cook et al., 2005; Weiss et al., 2005).

Table 3. Scaling factor $\beta$ for various processes associated with irrigation mosaics and positive, neutral or negative effect of irrigation mosaics versus one contiguous area

<table>
<thead>
<tr>
<th>Process</th>
<th>$\beta$</th>
<th>Irrigation mosaics versus contiguous area</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Advantage Neutral Disadvantage</td>
</tr>
<tr>
<td>Avoid adverse areas</td>
<td>not applicable</td>
<td>X</td>
</tr>
<tr>
<td>Exploit natural variability</td>
<td>not applicable</td>
<td>X</td>
</tr>
<tr>
<td>Evapotranspiration</td>
<td>-0.1 to 0</td>
<td>X X</td>
</tr>
<tr>
<td>Water table rise: Maximum height</td>
<td>0 to 0.35</td>
<td>X X</td>
</tr>
<tr>
<td>Water table rise: Area affected</td>
<td>0 to 0.57</td>
<td>X X</td>
</tr>
<tr>
<td>Solutes: Area affected</td>
<td>0 to 0.31</td>
<td>X X</td>
</tr>
<tr>
<td>Solutes: Concentration</td>
<td>not applicable</td>
<td>X X</td>
</tr>
</tbody>
</table>

Other tools in the form of new analytical solutions for water table rise under circular irrigation patches were developed and computer programs that considerably reduce the computation time were developed. These will prove useful when evaluation of development proposals is required. Along with these new solutions the mathematics to determine the effect of spacing and patch size for square and hexagonal arrays (mosaics) were developed. Here we have shown the effect for infinite arrays, and while the summations can be truncated for finite arrays, they will have to be modified to take account of the effect at the edges of the arrays.
These effects will only be noticeable when the non-dimensional time $\tau = \frac{\bar{b} K}{S_y R^2}$ is large. The non-dimensional time is the dimensional time ($t$) scaled with the physical parameters of the system; $K$ is saturated hydraulic conductivity, $\bar{b}$ is a linearisation parameter, $S_y$ is specific yield and $R$ is the radius of the irrigated patch. A computer program has been written for the square array, but there is need for further development of a program to compute the hexagonal array.

These array solutions could be exploited to investigate combinations of extraction wells and irrigated patches in mosaic patterns. These solutions would be very helpful in determining the amount of groundwater extraction that is possible, sighting options and potential effects on surface water flows. The solutions developed here have many possible future uses.

Periodic solutions can (we have checked that a solution can be developed) be developed given appropriate resourcing. These will provide the means to investigate the effect of the periodicity of the northern Australian climate on the results. They will also be useful for checking numerical models with homogeneous soil properties, when periodic inputs are used. This will help build confidence in the results when heterogeneous soil properties are used in these models.

The solutes have in this initial study been treated in a very simplistic manner. We have however also outlined a procedure to develop a particle tracking method for estimating solute transport. If combined with the array solutions this could provide more certainty in modelling solute transport in mosaics. We have not considered reactive solutes in this study so the tracer solutes used here provide an outer extreme to solute leakage from the irrigated area. More effort will be needed to address reactive solutes when considering specific locations.

The numerical results presented show similar effects to the analytical solutions but are able to include periodicity in the recharge. These show the same basic features with a rise in water table height with time and depending on the spacing, a mosaic arrangement can result in lower water table rise. Although not included here numerical results can allow for heterogeneity that is not easily included in analytical results. The two approaches together offer means of providing useful information when considering the design and management of an irrigation area involving a mosaic structure. The analytical solutions can be used to quickly assess various options and then more detailed results can be developed using the numerical results. This combination of methods should provide an efficient means of assessing various forms of irrigation mosaics.

The limited climate analysis provided here suggests that when considering the possibility for irrigation in northern Australia a detailed analysis should be undertaken, as although evaporation is relatively uniform, rainfall is not. This could result in annual surpluses of water in some years, but these surpluses may arise due to large events where the water is only in surplus for a short period of time.

The vexed question of advection as related to irrigated patch size is one that cannot be conclusively determined from this study. It is likely that the amount of advection will increase as both the patch size and number of patches decreases. There is a need to research this
in more depth especially for climatic conditions typical of northern Australia where consistent periods of dryness and high potential evaporation rates occur.

The results of this initial study demonstrate that there are, from a biophysical point of view, both advantages and disadvantages to irrigation mosaics, and that further work is needed to improve our level of understanding of irrigation mosaics and the actual benefits they could deliver. To achieve this there is particular need for further research and development of:

1. analytical solutions for periodic recharge of ground water
2. solute transport using particle tracing methods
3. improved understanding of finite arrays of mosaics
4. improved understanding of the effect advection has on evaporation rate as effected by size and number of irrigated patches, and
5. improved understanding of the water and salt balance and of irrigation requirements of sites across northern Australia.
9. Appendices

9.1. Appendix 1. Ground water Rise Solutions

9.1.1. Hantush 1967

The solutions for water table rise from rectangular and circular recharge areas due to Hantush (1967) are given below.

**Rectangle:**

The solution for water table rise is:

\[
h^2 - h_0^2 = vt(D_t / 2K)
\]

\[
s^*(\frac{b_h - x}{B_h}, \frac{a_h + y}{B_h}) + s^*(\frac{b_h - x}{B_h}, \frac{a_h + y}{B_h})
\]

\[
\int_0^1 \text{erf}(\frac{a_h}{\lambda}) \cdot \text{erf}(\frac{b_h}{\lambda}) d\tau \lambda
\]

Where

\[\nu = \frac{Kb}{S_y}\]

\[\bar{b} = \left( h_0 + h(t) \right)/2 \]

\[B = \sqrt{4\nu t}\]

\[t = \text{time} \ [T] \]

\[a_h = \text{half width of the rectangle (Fig. 4)} \ [L] \]

\[b_h = \text{half length of the rectangle (Fig. 4)} \ [L] \]

\[h_0 = \text{initial water table height} \ [L] \]

\[h(t) = \text{water table height at time} \ t \ [L] \]

\[S_y = \text{specific yield} \ [L^3 \ L^{-3}] \]

\[D_t = \text{vertical percolation rate} \ [L \ T^{-1}] \]

\[K = \text{saturated hydraulic conductivity} \ [L \ T^{-1}] \]

\[S^* \text{ is a function} \]

\[\lambda = \text{integrand} \]

The \(S^*\) function is nowadays easily solve numerically with computers rather than using the tables in Hantush. It should be noted that \(h(t)\) appears in \(\nu\), so the solution is iterative. However, when \(h(t) - h_0 \ll h_0\), \(\nu\) and hence \(\bar{b}\) will only be marginally effected by the \(h\).

The maximum height of the water table \((h_m)\) will occur at the point \(x = y = 0\) and is given by:

\[
h_m^2 - h_0^2 = vt(2D_t / K)s^*\left(\frac{b_h}{B_h}, \frac{a_h}{B_h}\right)
\]

\[\text{[A2]}\]
Circle:

The solution for water table rise under a circular recharge area is:

\[ h^2 - h_0^2 = \frac{2U}{\pi K} \int_0^\infty \left(1 - \exp(-q_b \delta^2)\right) J_1(\delta) J_0(\rho \delta) d\delta / \delta^2 \quad [A3] \]

\[ q_b = \frac{vt}{R^2}, \quad U = D_i \pi R^2, \quad \rho = \frac{r}{R} \]

where

- \( J_1 \) is a Bessel function of first kind and first order
- \( J_0 \) is a Bessel function of first kind and zero order
- \( \delta \) is the integrand

The maximum height

\[ h_m^2 - h_0^2 = \frac{U}{2\pi K \mu_0} \left[ E_1(\mu_0) + (1 - \exp(-\mu_0)) / \mu_0 \right] \]

\[ \mu_0 = R^2 / 4vt \quad [A4] \]

\[ E_1(x) = \int_x^\infty \exp\left(-\frac{\lambda}{\lambda}\right) d\lambda \]

Equation [A4] is the exponential integral of first kind or well function for non-leaky aquifers.

When \( t \geq 0.5R^2/v \) and \( r > R \) then eqn [A3] can be approximated by:

\[ h^2 - h_0^2 = \left(\frac{U}{2\pi K}\right) \left[ E_1(\mu) + 0.5 \mu_0 \exp(-\mu) \right] \quad [A5] \]

\[ \mu = \frac{r^2}{4vt} \]

Equation [A5] is what is of interest to us for determining the influence of the water table outside of the irrigated area. To do this we will assume that the \( h^2 - h_0^2 = \epsilon \) where \( \epsilon \) is some small value then eqn [A5] can be rearranged to give:

\[ \mu = \exp \left[ \frac{1}{2\mu_0} \left( \frac{2\epsilon K}{R^2} - E_1(\mu) \right) \right] \quad [A6] \]

Equation [A6] can be solved by recursive methods and \( \mu \) found. The value of \( r \) can then be obtained. The value of \( \Delta r = r - R \) and the relationship between \( \Delta r \) and \( R \) can be obtained.

Equation [A3] is not straightforward, as the Bessel functions are periodic and so convergence of the integral is slow. This was noted by Zotnik and Ledder (1991a), and they developed some useful criteria to assist with knowing when convergence is reached. One of us (J.H. Knight) developed a new solution based on Laplace Transforms which is given below.
New Solution:

For small $\tau (< 0.1)$ the function $f(\rho, \tau)$ in eqn [13] is given by:

$$f(\rho, \tau) = \tau - \frac{\tau}{\sqrt{\rho}} \left[ 2i^2 \text{erfc} \left( \frac{1 - \rho}{2\sqrt{\tau}} \right) \frac{1}{2} + \frac{(3 + 1/\rho)\sqrt{\tau}}{2} i^3 \text{erfc} \left( \frac{1 - \rho}{2\sqrt{\tau}} \right) ight.$$

$$- \frac{3(1 - 1/\rho)(5 + 3/\rho)\tau}{16} + 3(35 - 5/\rho + 9/\rho^3/25/\rho^4)\tau^{3/2} i^{4}\text{erfc} \left( \frac{1 - \rho}{2\sqrt{\tau}} \right)$$

$$+ \frac{15(1 - 1/\rho)(315 + 287/\rho + 305/\rho^2 + 245/\rho^3)\tau}{1024} i^{5}\text{erfc} \left( \frac{1 - \rho}{2\sqrt{\tau}} \right) \right]$$

$$, \rho < 1$$

where $i^n \text{erfc}(x)$ is the integral of the complimentary error function and can be calculated by (Carslaw and Jaeger, 1959, p483):

$$i^n \text{erfc}(x) = \text{erfc}(x) = 1 - \text{erf}(x)$$

$$i \text{erfc} = \frac{1}{\sqrt{\pi}} \exp(-x^2) - x \text{erfc}(x) \quad [A8]$$

$$2n i^n \text{erfc}(x) = i^{n-2} \text{erfc}(x) - 2x i^{n-1} \text{erfc}(x) \quad n = 2, 3, ...$$

For large $\tau (> 0.01)$ the function $f(\rho, \tau)$ is given by:

$$f(\rho, \tau) = \tau \left[ 1 - \exp \left( -\frac{1}{4\tau} \right) \right] + \frac{1}{4} E_1 \left( \frac{1}{4\tau} \right) \exp \left( -\frac{1}{4\tau} \right) \left( \frac{\rho^2}{4} + \frac{\rho^4}{256\tau^2} \right) + , \rho < 1$$

$$f(\rho, \tau) = \frac{1}{4} E_1 \left( \frac{1}{4\tau} \right) + \exp \left( -\frac{\rho^2}{4\tau} \right) \left( \frac{1}{32\tau} - \frac{1}{768\tau^2} + \frac{\rho^2}{3072\tau^3} \right), \rho > 1$$

[A9]
10. Glossary

\(a\) = half width of an elliptical or rectangular area (Figs 3, 4) [L]
\(a_n\) = width of rectangular recharge area [L]
\(a_e\) = empirical constant in eqn [24] [L^2]
\(A\) = major axis of single contiguous elliptical area [L]
\(A_e\) = parameter in wind function (eqn [24]) [M^{-1} L^{2.1} T]
\(A_o\) = area of an irrigated patch [L^2]
\(b\) = half length of an elliptical or rectangular area (Figs 3, 4)
\(b_n\) = empirical constant in eqn [24] [M^{-1} T]
\(b_h\) = length of rectangular recharge area [L]
\(\bar{b} = \frac{(h_o + h(t))}{2}\) is a linearization parameter in the Boussineqs eqn [L]
\(B\) = minor axis of large contiguous elliptical area [L]
\(B_n = \sqrt{4 \nu t}\) is a parameter in eqn [A1] [L T^{-1/2}]
\(c_a\) = empirical constant in eqn [24] [M^{-1} L^{-1} T^2]
\(C\) = solute concentration in liquid phase of soil [M L^{-3}]
\(C_{BR}\) = the concentration at the bottom of the rootzone [M L^{-3}]
\(C_0\) = the initial concentration at [M L^{-3}]
\(D\) = half-spacing of drains [L]
\(D_s\) = dispersion coefficient [L^2 T^{-1}]
\(D_t\) = deep percolation rate to [L T^{-1}]
\(e_a\) = saturated vapour pressure [M L^{-1} T^{-2}]
\(e_v\) = vapour pressure [M L^{-1} T^{-2}]
\(E\) = evaporation rate from a water body [L T^{-1}]
\(E_o\) = potential evaporation rate [L T^{-1}]
\(E_{To}\) = FAO56 potential evapotranspiration rate [M L^{-1} T^{-2}]
\(\exp(x) = \text{is the exponential function and } x \text{ is the argument}\)
\(\text{erf}(x) = \text{the error function and } x \text{ is the argument}\)
\(\text{erfc}(x) = \text{complimentary error function and } x \text{ is the argument}\)
\(f(u)\) = wind function [M^{-1} L^2 T]
\(f(\rho, \tau)\) = a function for the non-dimensional water table height
\(h_0\) = initial water table height [L]
\(h_m\) = maximum water table height [L]
\(h(t)\) = water table height at time \(t\) [L]
\(h|_t = h \text{ at time } t \text{ (eqn [21])} [L]\)
\(H\) = the non-dimensional water table height
\(H_D\) = the depth to drains [L]
\(H_m\) = the maximum non-dimensional water table height
\(i^n\text{erfc} = n\text{th integral of the complementary error function defined in eqn [A8]}\)
\(I\) = set of integers (eqn[18])
\(I_n\) = marginal impact of a single irrigated patch
\(I\) = marginal impact of contiguous irrigated area
\(I_{Re}\) = scaled marginal impact defined in eqn [8]
\(I_o\) = half length of the rectangle (Fig. 4) [L]
\(I_0\) = Bessel function of first kind and zero order
\(J_1\) = Bessel function of first kind and first order
\( J_b = \) Bessel function of first kind and zero order
\( K = \) saturated hydraulic conductivity \([L \ T^{-1}]\)
\( L = \) spacing between irrigation patches in mosaic arrays \([L]\)
\( n = \) number of irrigated patches in mosaic
\( P = \) perimeter of and area \([L]\)
\( q = \) Darcian flux of water \([L \ T^{-1}]\)
\( q_h = \frac{vt}{R^2} \) non-dimensional flux in eqn \([A3]\)
\( R = \) radius of circular irrigated patch \([L]\)
\( R_s = \) the retardation factor for solutes
\( R_0 = \) radius to which solute has advected in time \( t \) \([L]\)
\( R_T = \) radius of circular contiguous irrigated area \([L]\)
\( S^* = \) a function defined in eqn \([A1]\)
\( S_y = \) specific yield \([L^3 \ L^{-3}]\)
\( t = \) time \([T]\)
\( t_s = \) time for travel of solute from root zone to ground water \([T]\)
\( t_w = \) travel time for water from rootzone to water table \([T]\)
\( U = D_\pi R^2 \) is flow rate in eqn \([A3]\) \([L^3 \ T^{-1}]\)
\( V_i = \) depth of water added to the water table in time \( t \) \([L]\)
\( V_s = \) depth of water stored in the water table in time \( t \) \([L]\)
\( x = L/2 \) variable used in eqn \([23]\) \([L]\)
\( x_h = x \) axis for rectangular recharge area \([L]\)
\( y = L/3 \) \([L]\)
\( y_h = y \) axis for rectangular recharge area \([L]\)
\( z_{wt} = \) depth from the surface to the water table \([L]\)
\( z_{RT} = \) depth from the surface to the base of the rootzone \([L]\)
\( \alpha = \) is the dummy variable in eqn \([14]\)
\( \alpha_c = \) the decay constant
\( \alpha_h = \) dummy variable in \( S^* \) function (eqn[A1])
\( \alpha_x = \) constant in scaling relationship, \( x \) is either \( a \) or \( b \) (eqn \([7]\))
\( \beta_h = \) dummy variable in \( S^* \) function (eqn[A1])
\( \beta_x = \) power constant in scaling relationship, \( x \) is either \( a \) or \( b \) (eqn \([7]\))
\( \varepsilon = \) a small increment of \( h \) \([L]\)
\( \delta = \) an integrand in eqn \([A3]\)
\( \Delta a = \) length of impacted area on major axis of elliptical area \([L]\)
\( \Delta b = \) length of impacted area on minor axis of elliptical area \([L]\)
\( \Delta A = \) length of impacted area on major axis of contiguous elliptical area \([L]\)
\( \Delta B = \) length of impacted area on major axis of contiguous elliptical area \([L]\)
\( \Delta h(r) \big|_0 = h(r) \big|_v \) is the value of the integrand in eqn \([21]\) \([L]\)
\( \Delta H = \) contribution of water table change due to drains \([L]\)
\( \Delta r = \) radial distance of impact of irrigation on surrounding area \([L]\)
\( \gamma = \) constant in eqn \([5]\)
\( \lambda = \) integrand (eqn[A1, A4])
\( \mu = r^2 / 4vt \) is a non-dimensional variable in eqn \([A5]\)
\( \mu_0 = R^2 / 4vt \) is a non-dimensional variable in eqn \([A4]\)
\( \nu = K\beta / S_y \) parameter in eqn[A1] \([L^2 \ T^{-1}]\)
\( \rho = \) non-dimensional radius
\( \tau = t_0^2 / (S_y R^2) \) is the non-dimensional time
\[ v = \frac{K \theta}{S_y} \quad [L^2 \ T^{-1}] \]

\( \theta \) = water content of soil or aquifer \([L^3 \ L^{-3}]\)

\( \theta_s \) = saturated water content of soil or aquifer \([L^3 \ L^{-3}]\)

\( \theta_i \) = the water content in the vadose zone behind the wetting front \([L^3 \ L^{-3}]\)

\( \Delta \theta \) = the change in water content of the vadose zone \([L^3 \ L^{-3}]\)

\( \nabla \) = div mathematical operator
11. References


