Plant water uptake at the single plant scale: experiment vs. model

David Matthew Deery
Graduate Certificate in Mathematics, Charles Sturt University
Bachelor of Applied Science Agriculture Honours, Charles Sturt University
Bachelor of Applied Science Agriculture, University of Melbourne

A thesis submitted for the degree of Doctor of Philosophy at Charles Sturt University

August, 2008
Abstract

The purpose of this study is to determine what is limiting the extraction by wheat roots of seemingly available water in the subsoil. The agronomic experience is that subsoil water may contribute significantly to final yield, but that the ability of the roots to extract the water varies from crop to crop in ways that are not well understood.

The literature revealed several hypotheses as to why the extraction is incomplete. This study has focused on explicitly testing one of these hypotheses: that the soil is the main resistance to the extraction of water by the plant roots, owing to a combination of low root length density (unit length of root per unit volume of soil), low soil water diffusivity at low soil water content.

To test this hypothesis wheat plants were grown in undisturbed and repacked field soil comprising two common soil types used for cropping, namely undisturbed and repacked clay-loam from south-eastern Australia and repacked sand from Western Australia. The plants were kept in a controlled environment where they were challenged with a range of evaporative demands, first rising and then falling, and the transpiration rate, $E$, and the null measurement of the xylem water potential, $B$, were measured non-destructively and continuously. The experimental measurements were compared to the output of a mathematical model that solves the radial diffusion equation for the flow of water to a single plant root, assumed to represent all roots.

An important function for modelling the flow of water to a plant root is the soil water diffusivity in the range from 100 to 1500 kPa suction. A method was developed for measuring the soil water diffusivity in this range, that can also be used on undisturbed soil.

For all soil types, during the rising phase of $E$, $B$ as a function of $E$, $B(E)$, was linear at low to moderate $E$, with a constant slope that represented the hydraulic resistance of the plant. However at high $E$, $B$ accelerated with $E$. For the repacked clay-loam and the repacked sand, during the falling phase of $E$, $B(E)$ was essentially linear over the
whole range of $E$. This was in contrast to the undisturbed clay-loam, where $B$ often curved downwards with decreasing $E$.

For the repacked clay-loam and the repacked sand, the model could match the data during the rising phase of $E$, if it was assumed that only 10% of the roots were taking up water and that the soil water diffusivity was constant and low. However it could not match the data during the falling phase of $E$, unless it was assumed that there had been a significant rise in the hydraulic resistance of the plant, or perhaps more likely, that an additional, yet constant, interfacial resistance had developed when $E$ was high and $B$ was rapidly increasing. For the sand, the postulated interfacial resistance was about the same size as that within the plant and, for the clay-loam, the postulated interfacial resistance was about a quarter of that within the plant.

That the slope of $B(E)$ during the falling phase of $E$, for the repacked clay-loam and the repacked sand, was essentially constant suggests that the radial flow of water through the soil generated only minor gradients in soil suction and therefore that neither low soil water diffusivity nor low root length density was inhibiting the extraction of water from the soil by the plant roots.

For the undisturbed clay-loam soil, the radial-flow model did not agree with the experimental data even when various combinations of soil water diffusivity and root length density were tried. This disagreement may have been due to a skewed distribution of roots in the cores, for few if any roots were seen at the base of any core. However, even when the roots were assumed to be confined to the top 50 or 75% of the total soil volume the model and experimental data did not agree.

This work provides evidence that the flow of water to the plant roots, as encapsulated in the radial-flow model, is not inducing large gradients in suction close to the plant roots growing in the three soil types used in the experiments reported here. The clear disagreement between the experimental data and the model suggests that something else is generating the large hydraulic resistances evident between the soil and the leaves of the plants.
Acknowledgements

Firstly I thank my principal supervisor, John Passioura, who despite retirement status proved anything but and continually challenged me to do better. His vast and extensive technical expertise, guidance, dedication and patience is gratefully acknowledged here. I am grateful for the guidance provided by my university supervisors, Jason Condon and Asitha Katupitiya.

I am grateful to the Cooperative Research Centre for Irrigation Futures for awarding me a PhD scholarship, without which this project would not have been possible. I am also grateful for the collegiate nature and network of the Cooperative Research Centre for Irrigation Futures that provided an extra tier of support. I am grateful to the CSIRO Water for Healthy Country flagship, for generously providing a PhD top up scholarship and operating funds. I am also grateful for support from the EH Graham Centre at Charles Sturt University.

I thank my many colleagues at CSIRO Plant Industry Crop Adaptation. I have benefited greatly from many stimulating discussions and cherish the friendships that I have gained in this stimulating environment.

I must also thank my friends for their unconditional friendship. In particular I am indebted for the support and understanding provided by my partner, Fi.

Finally, I am forever indebted to my parents, Kevin and Veronica, and my siblings Rob, Chris, Bridget and Simon for their understanding and support.
Commonly used nomenclature

$B$  null measurement of the xylem water potential equal to the pressure drop across the plant and soil, termed balancing pressure [kPa]

$B^*$  half distance between biopores [m]

$D(\theta)$  soil water diffusivity as a function of soil water content [m$^2$ s$^{-1}$]

$D$  soil water diffusivity [m$^2$ s$^{-1}$]

$D_{ref}(\theta)$  soil water diffusivity at reference temperature

$D_T(\theta)$  soil water diffusivity at temperature T

$E$  plant transpiration rate [m$^3$ s$^{-1}$], [g s$^{-1}$] or [µg s$^{-1}$].

$F*$  flux density of water per unit cross sectional area [m$^3$ m$^{-2}$ s$^{-1}$]

$F$  flux of water [m s$^{-1}$]

$F_{air}$  flow rate of air through the cuvette containing the plant [m$^3$ s$^{-1}$]

$g$  stomatal conductance [mol m$^{-2}$ s$^{-1}$]

$I$  electrical current [Amperes]

$i$  time point when two different calculations of the quantity of water lost from one-dimensional soil profile by evaporation or quantity of water taken up by the root from the cylinder of soil, of unit height, defined by $r_a$ and $r_b$, are compared

$k$  soil hydraulic conductivity [m s$^{-1}$]

$L$  length of sample [m]

$L_v$  root length density (length of root per volume of soil) [cm cm$^{-3}$]

$L_v^*$  length of root occupied biopore per volume of soil [cm cm$^{-3}$]

$LA$  leaf area

$M_m$  is the molar mass of water [g mol$^{-1}$]

$m$  shape coefficient for soil water retention curve

$N$  number of replicate samples used

$n$  particular node in finite difference grid and shape coefficient for soil water retention curve

$O$  rate of outflow (Passioura $D(\theta)$ analysis)

$P_{atm}$  atmospheric pressure (assumed to be 1013 hPa),

$P_{diff}$  difference in vapour pressure between ingoing and outgoing air-streams from the cuvette [hPa]
\( p_s \) saturation pressure of liquid [hPa] water at steam point temperature, \( T_s \)

\( p_w \) partial pressure of water vapour [hPa]

\( p_0 \) saturation vapor pressure over liquid water [hPa] at given temperature

Note that ratio of \( p_w \) to \( p_0 \) is the relative humidity

\( \Delta P_{Plant} \) pressure drop across the plant [kPa]

\( Q \) rate of extraction of water from the soil \([m^3 \cdot m^{-3} \cdot s^{-1}]\)

\( R \) universal gas constant: 8.31 \([J \cdot K^{-1} \cdot mol^{-1}]\)

\( R_{Plant} \) resistance to flow within the plant \([kPa \cdot s \cdot \mu g^{-1}]\)

\( r \) radial distance from the root surface \([m]\)

\( r_a \) radius of the root \([m]\)

\( r_b \) radius of the outer boundary that the root is assumed to have exclusive access \([m]\)

\( S S_R \) sum of the squared residuals

\( svol \) volume of the pot containing soil and plant roots \([m^3]\)

\( T \) absolute temperature \([K]\)

\( T_{ref} \) reference temperature

\( T_s \) steam point temperature \([373.16^0K \text{ at one atmosphere } = 1013.246 \text{ hPa}]\)

\( t \) time \([s]\)

\( t^{1/2} \) square root time \([s^{1/2}]\)

\( V \) voltage \([\text{volts}]\)

\( V_m \) molar volume of an ideal gas at the given temperature \([m^3 \cdot mol^{-1}]\)

\( W \) quantity of water lost from the one-dimensional soil profile by evaporation \([m]\) or quantity of water taken up by the root from the cylinder of soil, of unit height, defined by \( r_a \) and \( r_b \) \([m^3]\) and amount of water remaining in the soil (Passioura \( D(\theta) \) analysis)

\( W_0 \) initial quantity of water in the one-dimensional soil profile \([m]\) or initial quantity of water in the cylinder of soil defined by \( r_a \) and \( r_b \) \([m^3]\)

\( x \) spatial position along length of sample \([m]\)

\( z \) depth \([m]\)

\( \alpha \) inverse of air-entry value \([\text{kPa}^{-1}]\) for soil water retention curve

\( \theta \) soil water content \([m^3 \cdot m^{-3}]\)

\( \theta_a \) soil water content at root surface \([m^3 \cdot m^{-3}]\)
\( \theta_b \) soil water content at the outer boundary \([\text{m}^3 \text{ m}^{-3}]\)

\( \theta_i \) initial soil water content \([\text{m}^3 \text{ m}^{-3}]\)

\( \theta_f \) final soil water content \([\text{m}^3 \text{ m}^{-3}]\)

\( \theta_L \) soil water content at \( x = L \) \([\text{m}^3 \text{ m}^{-3}]\)

\( \theta_r \) residual soil water content \([\text{m}^3 \text{ m}^{-3}]\) for soil water retention curve

\( \theta_s \) saturated soil water content \([\text{m}^3 \text{ m}^{-3}]\) for soil water retention curve

\( \lambda \) Boltzmann transform \([\lambda = x^{-2}]\)

\( \mu_{\text{ref}} \) dynamic viscosity at reference temperature

\( \mu_T \) dynamic viscosity at temperature \( T \)

\( \rho_w \) the density of liquid water \([\text{g} \text{ cm}^{-3}]\)

\( \rho_T \) density of soil water at reference temperature

\( \rho_{\text{ref}} \) density of soil water at temperature \( T \)

\( \sigma_{\text{ref}} \) surface tension at reference temperature

\( \sigma_T \) surface tension at temperature \( T \)

\( \tau \) soil matric suction \([\text{kPa}]\)

\( \Delta \tau \) suction drop across the root surface and the bulk soil \([\text{kPa}]\)

\( \tau_{\text{Bulk}} \) soil suction in bulk soil \([\text{kPa}]\)

\( \tau_{\text{ref}} \) soil suction at reference temperature

\( \tau_T \) soil suction at temperature \( T \)

\( \psi \) soil water potential

\( \Omega \) electrical resistance \([\text{Ohms}]\)
# Table of contents

Chapter 1  General introduction ................................................................................. 1
  1.1  Summary ........................................................................................................... 1
  1.2  The agronomic issue ........................................................................................ 1
  1.3  Possible reasons for incomplete extraction ..................................................... 4
      1.3.1  Soil limiting .............................................................................................. 4
      1.3.2  Experimental testing of the single root model ........................................... 6
      1.3.3  Root clumping ......................................................................................... 8
      1.3.4  Root shrinkage ....................................................................................... 8
      1.3.5  Osmotic pressure at the root surface ......................................................... 11
      1.3.6  Hydraulic resistance of the plant ............................................................... 11
  1.4  General plant water uptake models ................................................................. 12
  1.5  Hypotheses for what is limiting uptake of water by plant roots .................. 14

Chapter 2  Method development for measurement of soil water diffusivity ............. 16
  2.1  Introduction ..................................................................................................... 16
      2.1.1  Theory .................................................................................................... 18
  2.2  Evaporation experiment .................................................................................... 20
  2.3  Modelling evaporation ..................................................................................... 27
  2.4  Optimization of soil water diffusivity function ............................................... 32
  2.5  Summary ......................................................................................................... 35

Chapter 3  Measurement of soil water diffusivity on undisturbed and repacked soil 37
  3.1  Introduction ..................................................................................................... 37
  3.2  Method .............................................................................................................. 37
      3.2.1  Evaporation experiment .......................................................................... 37
      3.2.2  Measurement of hydraulic conductivity and soil water retention ...... 38
      3.2.3  One-step outflow experiment .................................................................. 40
      3.2.4  Analysis of one-step outflow data ........................................................... 42
  3.3  Results and discussion ..................................................................................... 42
      3.3.1  Hydraulic conductivity and soil water retention for determining soil water diffusivity on undisturbed soil at 8 kPa suction .............. 42
      3.3.2  Evaporation method for measurement of soil water diffusivity on undisturbed soil ................................................................. 44
3.3.3 Evaporation method for measurement of soil water diffusivity on repacked clay-loam ................................................................. 50
3.3.4 Evaporation method for measurement of soil water diffusivity on repacked sand ................................................................. 54
3.3.5 Outflow method for measurement of soil water diffusivity on undisturbed soil ................................................................. 57
3.4 Summary and discussion ................................................................. 61
  3.4.1 Undisturbed clay-loam ................................................................. 61
  3.4.2 Repacked clay-loam ................................................................. 64
  3.4.3 Repacked sand ................................................................. 65
  3.4.4 Conclusion ............................................................................ 67

Chapter 4 Method for simultaneous measurement of plant water uptake and pressure drop across the plant and soil ................................................................. 68
  4.1 Introduction ............................................................................ 68
  4.2 Pressure control of root chamber ................................................................. 70
  4.3 Adjusting pressure in root chamber according to leaf xylem water potential 74
  4.4 Regulating air and nitrogen supply to root chamber ................................................................. 78
  4.5 Atmospheric conditions inside cuvette ................................................................. 78
  4.6 Temperature control and measurement ................................................................. 81
  4.7 Logging outputs and measurement of transpiration ................................................................. 84
  4.8 Summary ............................................................................ 87

Chapter 5 Plant water uptake: experiment versus model ................................................................. 88
  5.1 Introduction ............................................................................ 88
  5.2 Methods ............................................................................. 92
    5.2.1 Plant water uptake experiment ................................................................. 92
    5.2.2 Plant water uptake model ................................................................. 98
    5.2.3 Soil hydraulic properties ................................................................. 102
  5.3 Results and discussion ................................................................. 103
    5.3.1 Undisturbed clay-loam ................................................................. 103
    5.3.2 Repacked clay-loam ................................................................. 118
    5.3.3 Repacked sand ................................................................. 125
  5.4 Conclusion ............................................................................ 135

Chapter 6 General Discussion ................................................................. 139
Certificate of Authorship

DAVID MATTHEW DEERY

Hereby declare that this submission is my own work and that, to the best of my knowledge and belief, it contains no material previously published or written by another person nor material which to a substantial extent has been accepted for the award of any other degree or diploma at Charles Sturt University or any other educational institution, except where due acknowledgment is made in the thesis. Any contribution made to the research by colleagues with whom I have worked at Charles Sturt University or elsewhere during my candidature is fully acknowledged.

I agree that the thesis be accessible for the purpose of study and research in accordance with the normal conditions established by the University Librarian for the care, loan and reproduction of the thesis.*

X

Signature .................................................. Date

* Subject to confidentiality provisions as approved by the University.
Chapter 1 General introduction

1.1 Summary
Subsoil water extracted by wheat crops during grain-filling contributes greatly to their yield. Often though, the available water in the subsoil is not fully utilized, even though seemingly adequate number of roots may be there. Several hypotheses have been identified in the literature about why the extraction is incomplete. These include: that the flow rate of water through the soil to individual, though well-distributed roots, is limited by the hydraulic properties of the soil; that there is a large interfacial resistance to the flow of water between the soil and root, possibly exacerbated by root shrinkage and vapour gaps; that the roots are clumped, so that water must move long distances to them; and that there may be an osmotic choke if solutes excluded by the root as the water enters increase in concentration at the root surface. There have been many attempts to test these hypotheses, with varying degrees of success, though none has dealt with roots growing in undisturbed field soils. Root water uptake is described in crop simulation models both mechanistically, as the radial flow of water to individual roots, and empirically using various types of sink terms.

1.2 The agronomic issue
The extraction of water from the subsoil by wheat crops during grain-filling is especially valuable to grain yield (Kirkegaard et al. 2007; Manschadi et al. 2006; Angus & Herwaarden 2001). Kirkegaard et al. (2007) found that such water (used after flowering) increased grain yield by 0.6 t ha\(^{-1}\), which represented an efficiency of 59 kg of grain per ha per mm of water used. This is nearly three times the well-established benchmark of wheat grain produced per mm of water used throughout the growing season in southeastern Australia, namely 20 kg ha\(^{-1}\) mm\(^{-1}\) (French & Schultz 1984; Sadras & Angus 2006). However Kirkegaard et al. (2007) found that despite the appreciable efficiency of subsoil water use, the crop failed to extract all of the water seemingly available to it. Clearly crops that extract more subsoil water are likely to yield better (Jordan & Miller 1980; Passioura 1983).

Plant available water is typically referred to as that held between the soil water content held at 1500 kPa suction for “lower limit” and the soil water content when the soil has practically ceased draining after soaking rain, known as “field capacity” or
“upper limit” (Ritchie 1981). The water held between these limits is then summed up over the estimated rooting depth of the soil profile to give the total amount of available water. These limits are useful but they are also arbitrary. The definition of field capacity implies that the soil actually stops draining; however, the soil will continue to drain indefinitely unless some equilibrium is reached and also the plant can use water held at suctions higher than that equal to “field capacity”. Lower limit is arbitrarily defined as 1500 kPa suction, yet plants can continue to extract water from suctions greater than 1500 kPa and plant growth can be restricted before the 1500 kPa suction is reached (Ritchie 1981). However, investigation has shown that the choice of 1500 kPa suction is generally appropriate for water balance studies (Savage et al. 1996). Although it may be useful to conceptualize the amount of water available to the crop as static, the determination of the actual upper and lower limits shows that the amount of available water is dynamic.

The literature contains examples of both annual crops drying, and failing to dry, the soil to 1500 kPa, some of which are discussed in Fischer (1979) and Passioura (1983). Some examples of wheat crops failing to dry the soil to 1500 kPa were reported by Schultz (1971), Schultz (1972), Hurd (1974), Walter & Barley (1974), Jordan & Miller (1980) and most recently Kirkegaard et al. (2007). In contrast, some instances of wheat crops drying the soil to a suction of 1500 kPa were presented by Power, Grunes & Reichman (1961), Fischer & Kohn (1966), Kohn & Storrier (1970) and Angus & Herwaarden (2001). Interestingly, Angus & Herwaarden (2001) found in their field experiment that at maturity the wheat crop supplied with no nitrogen dried the soil to a suction of 3000 kPa. Whereas the wheat crop supplied with 80 kg ha⁻¹ of nitrogen dried the soil to a suction of 5000 kPa, extracting an extra 15 mm of water which, according to the authors, is sufficient to contribute significantly to yield.

Recently a simulation study using historic weather data (Lilley & Kirkegaard 2007) showed that subsoil water will be more valuable in higher rainfall environments, owing to its more frequent occurrence. The study used the APSIM wheat crop simulation model which was calibrated using field experimental data from near Bethungra, NSW, for 11 rain-fed wheat crops and a wheat crop grown under a rainout shelter. The simulation study used 106 years (1900-2005) of weather data to calculate wheat yield and water use for three different locations under two different treatments: (1) a dry treatment, comprising a wheat crop sown into a dry profile following
removal of a lucerne pasture that had fully dried the soil profile to 1.8 m and (2) a wet treatment, comprising a wheat crop sown into a wet soil profile following a wheat crop that had only extracted water to 1.2 m. The three sites (located 22 km east, 25 km west and 96 km west of Bethungra) differed in mean annual rainfall by 140 mm. The soil profile in the dry treatment did not wet below 1.2 m for 80%, 70% and 38% of the years for the three locations. However, for 30% of the years at the location with the greatest rainfall, the grain yield difference between the dry and wet treatments was 0.8-1.0 t ha\(^{-1}\). In years when the subsoil water was used, the marginal water use efficiency (defined as \([\text{yield of 1.8 m treatment} - \text{yield of 1.2 m treatment}] / [\text{water uptake from the 1.2-1.8 m layer}]\)) for the majority of seasons in the wet treatment was 30-50 kg ha\(^{-1}\) mm\(^{-1}\). For the dry treatment, the marginal water use efficiency was 30-50 kg ha\(^{-1}\) mm\(^{-1}\) for the wettest location in about 35% of the seasons.

As well as the demonstrated production benefits of subsoil water use (Kirkegaard et al. 2007), increasing crop water extraction from deep in the root zone can decrease the likelihood of deep drainage that can result in other deleterious environmental effects (Dunin & Passioura 2006). O'Connell, O'Leary & Connor (2003) showed that drainage of water below the subsoil can be reduced, in the Victorian Mallee region, by replacing the fallow with mustard in a three-year rotation comprising wheat and pea. Recent reviews have considered both agronomic and genetic methods to increase subsoil water use (Tennant & Hall 2001; Passioura 2006; Gregory 2006). Gregory (2006) discussed plant breeding and variety selection methods to increase rooting depth. He also discussed the use of agronomic practices, such as incorporating deep-rooted perennial pasture plants like lucerne into crop rotations, which may increase the rooting depth of following wheat crops. Tennant & Hall (2001) reported gains of 40-70 mm year\(^{-1}\) of extra water extraction on deep sandy soils and loamy sand soils from Western Australia using deep-rooted and longer growing crop and pasture species and from amelioration of traffic pans and subsoil acidity and/or selection of tolerant species. Passioura (2006) discusses agronomic and plant breeding methods to increase the water productivity (grain yield per water used). In situations where water is the limiting resource there is potential for matching crop development better to the pattern of water supply by reducing soil evaporation and maintaining a good balance of water-use prior and post flowering. Also where wheat grain yield is below potential (20 kg ha\(^{-1}\) mm\(^{-1}\) for south eastern Australia, see French & Schultz (1984); Sadras & Angus (2006)) there is scope to reduce the impact of other stresses such as diseases,
weeds, poor nutrition and hostile soil conditions. These reviews suggest that there is potential for improving crop water use and crop productivity through agronomic methods like crop rotations, zero till, and using deep-rooted cultivars with longer growing seasons.

1.3 Possible reasons for incomplete extraction

1.3.1 Soil limiting

One hypothesis for incomplete extraction of all the available subsoil water, derived from the analysis of Philip (1957c), is that the flow of water to the plant roots is limited by the soil hydraulic properties. Philip solved the cylindrical diffusion equation, eq. 1, to describe the radial flow of water through the soil to a plant root

\[
\frac{\partial \theta}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( rD \frac{\partial \theta}{\partial r} \right)
\]

(1)

where \( \theta \) is the volumetric soil water content [m\(^3\) m\(^{-3}\)], \( t \) is time [s], \( r \) is radial distance from the root [m] and \( D \) is soil water diffusivity [m\(^2\) s\(^{-1}\)]. Generally, when using this approach, it is assumed that each root has sole access to a hollow cylinder of soil with inner radius, \( r_a \), equal to the root radius and outer radius, \( r_b \), determined from the root length density [cm cm\(^{-3}\)], \( L_v \), using eq. 2. This is known as the single root model.

\[
r_b = \frac{1}{\sqrt{\pi L_v}}
\]

(2)

The analysis by Philip (1957c) solved eq. 1 analytically with a constant \( D \), a zero flux boundary condition at \( r_b \) and a flux boundary condition at the root surface. Philip concluded that treating “wilting point” (he defined “wilting point” as a suction of 2.5 MPa) as a constant lower limit of plant available water could be misleading. The analysis demonstrated that the mean soil water content when 2.5 MPa suction was reached at \( r_a \), was dependent on transpiration rate and root geometry (\( r_a \) and \( r_b \)).

Gardner (1960) derived a simple solution of eq. 1 by considering steady state conditions, \( \frac{\partial \theta}{\partial t} = 0 \), and constant \( D \). An approximate solution for transient plant water uptake was then obtained by assuming a succession of steady states. The steady state assumption means that over any given time period all the water taken up by the root at \( r_a \) is matched by an influx of water at \( r_b \).
Cowan (1965) adapted the model of Philip (1957c) and Gardner (1960) and obtained an analytical solution to eq. 1 assuming constant $D$, and, like Philip, a zero flux boundary condition at $r_b$, thereby assuming that the water extracted by the root comes from the volume of soil defined between $r_a$ and $r_b$. Cowan used a steady rate, $(\delta\theta/\delta t = \text{constant})$, approximation to illustrate the large gradients in soil water content and soil water potential that could occur within mm of the root. He thereby demonstrated that measurement of the bulk soil water status would provide little indication of the water status at the root-soil interface and the plant. In contrast, Gardner (1960) assumed that the water extracted by the root was supplied into the volume of soil defined between $r_a$ and $r_b$, at $r_b$.

A critique of the Gardner and Cowan models (Newman 1969) suggested that the values of $L_v$ they assumed were much less than those commonly encountered in the field, which resulted in an over estimate of rhizosphere resistance. Cowan (1965) considered four $L_v$s; 0.5, 0.25 0.125 [cm cm$^{-3}$] and ‘very dense rooting’. In a field experiment, Kirkegaard et al. (2007) found that the mean $L_v$ decreased from 10 [cm cm$^{-3}$] close to the soil surface to 1 [cm cm$^{-3}$] at 0.8 m depth to 0.1 [cm cm$^{-3}$] at 1.4 m depth and finally to 0.01 [cm cm$^{-3}$] at 1.8 m depth, indicating that the values of $L_v$ chosen by Cowan (1965) are representative of those encountered in the field.

Passioura & Cowan (1968) compared predictions of soil water content at the soil-root interface from approximate steady state and steady rate solutions with a numerical solution of eq. 1. The approximate solutions agreed adequately with the numerical solution, with the steady rate solution more accurate than the steady state in terms of closeness to the numerical solution.

This early work, comprehensively reviewed by Tinker (1976), in effect tests the hypothesis that flow through the soil becomes limiting to the extraction of water by the plant roots from the soil as the soil dries, for soil water diffusivity decreases to such a small value such that the soil can no longer meet the requirement of the root. Although it is well established that the greatest resistance by far, in the transpiration path from the soil to the atmosphere, is at the stomata.
1.3.2 Experimental testing of the single root model

Various experimental attempts have been made to test the hypothesis that the soil hydraulic properties limit the extraction of water by plant roots from the soil.

Lang & Gardner (1970) compared calculations of the flux of water to a plant root, assuming steady state conditions, with experimental measurements of the total root length and transpiration rate of cotton plants. The calculated flux was three orders of magnitude greater than the measured flux: they concluded that the roots have some areas that do not absorb water or that there is sufficient resistance between the root surface and the xylem such that the measured water potential in the xylem is much lower than that at the root surface. They measured the water potentials of the xylem in the plant and the soil continuously and in situ using psychrometers.

Herkelrath, Miller & Gardner (1977a) grew wheat plants in containers with the soil sectioned horizontally with wax membranes, permeable to roots but not water. The objective was to measure the vertical distribution of root water uptake and test the theoretical model of Gardner (1960). The leaf water potential was measured using the in situ leaf hygrometer technique described by Baughn & Tanner (1976). An estimate of the root water potential was made by allowing a thin section of soil or “test section”, sectioned with wax membranes, to equilibrate with the surrounding soil, while the rest of the soil was maintained wet. The water potential of the thin section of soil was then measured. They found that despite the soil suction of 86% of the root zone soil being wetter than 10 kPa, the roots dried the “test section” to a suction of 500 kPa. They then inferred a root suction of 500 kPa and a measured leaf suction of also 500 kPa. They thus assumed that there was little resistance to flow between the roots and the leaves and that the resistance to flow between the soil and the root xylem was large. The root water extraction rate decreased rapidly when the soil suction dropped lower than 10 kPa. Although plants in the field typically have little trouble extracting water from the soil at 10 kPa suction, Herkelrath, Miller & Gardner (1977a) used a sand in their experiment which can have little plant-available water, even at 10 kPa suction.

Herkelrath, Miller & Gardner (1977b) compared the experimental results in Herkelrath, Miller & Gardner (1977a) to those predicted by root uptake models. They found that the extraction rates predicted by the theory were as much as eight times
greater than the measured values. To obtain an agreement between experimental results and theory they had to assume a rooting density of 100 times smaller than that measured, thereby suggesting that less than the total measured root length was extracting water.

Herkelrath, Miller & Gardner (1977b) then proposed an alternative model, called the root contact model. This model assumed that as the soil dried, the root surface area in contact with the soil decreased, which increased the root interfacial resistance. The effective root membrane permeability was assumed to be proportional to the relative saturation of the soil. The root contact model gave a much better fit to the experimental data in Herkelrath, Miller & Gardner (1977a) than the conventional model of Gardner (1960).

Bristow, Campbell & Calissendorff (1984) investigated the effects of soil texture on the resistance to water movement within the rhizosphere on sunflower plants grown in pots. They measured transpiration rate, soil and xylem suctions, rooting density and the soil hydraulic properties. The total resistance to water movement increased most rapidly in the coarse textured soil, and least in the fine textured soil. The increase in hydraulic resistance as the soil dried was attributed to an increase in interfacial resistance between the root and soil.

Passioura (1980) simultaneously measured the pressure drop across the plant and soil and the plant transpiration rate. The experimental results were compared to a model that assumed: withdrawal of water from the soil by the root at a constant rate (i.e. $\frac{\partial \theta}{\partial t} = \text{constant}$), that each root had sole access to a hollow cylinder of soil with inner radius, $r_a$, equal to the root radius and outer radius, $r_b$, determined from $L_v$ using eq. 2 and a zero flux boundary condition at $r_b$. He found the model and experimental results agreed if about 30% of the measured root length was effective in taking up water. The plants were grown in a fertile loam and agreement between experiment and model was attained without accounting for any interfacial resistance. This is in contrast to Bristow, Campbell & Calissendorff (1984) where a sand and two silt loams were used and the interfacial resistance increased as the soil dried: the difference in interfacial resistance between Passioura (1980) and Bristow, Campbell & Calissendorff (1984) may be due to soil textural differences.
1.3.3 Root clumping

Some of the complications pertaining to the underlying model, eq. 1, and why it may fail to accurately describe the uptake of water by roots are reviewed by Passioura (1988). These complications include roots growing in pre-existing biopores and clumping, where the clumped roots growing in a biopore effectively act as a large single root. As well as clumping, roots growing in large diameter pores may have poor contact with the soil. This is in contrast with a root creating its own pore, and having intimate contact with the soil. In the former case, a proportion of water may flow as vapour resulting in a lower flux of water to the root.

Passioura (1991) used the single root model applied to roots clumped in biopores to estimate the time constant for roots growing in biopores to extract the available water. Using a simple characteristic time analysis, based on solutions of eq. 1, he estimated the time constant to be of order $B^2/D$, where $B^*$ is the half distance between biopores ($B^* = l^/l\pi^* \approx = L^*/\pi$ where $L^*$ is the length of occupied biopore per volume of soil) and $D$ is assumed to be at a minimum value of approximately 1 cm$^2$ day$^{-1}$, which pertains across a broad range of soil types (Rose 1968). The time constant was estimated to be of the order of weeks, if the distance between the biopores was large (e.g. 10 to 20 cm). This is compared to three days for roots evenly distributed in soil with a low root length density of 0.1 [cm cm$^{-3}$].

1.3.4 Root shrinkage

Some researchers have speculated that root shrinkage, where the diameter of the root decreases, may increase the resistance to flow from the soil to the xylem vessel inside the root. Shrinkage causes part of the root surface to come away from the soil, resulting in the development of a gap at the soil-root interface, where vapour flow, as opposed to liquid flow, is then supposedly the dominant mode of water transport. Root shrinkage is embodied in the “root contact model” of Herkelrath, Miller & Gardner (1977a).

Faiz & Weatherley (1977) found experimentally that a large hydraulic resistance developed in the rhizosphere of rapidly transpiring plants. In a further paper (Faiz & Weatherley 1978) they hypothesized that the high resistance to flow was located at the soil-root interface. This resistance was thought to increase as a result of contraction of
water stressed roots, resulting in the development of vapour gaps between the root and soil.

Faiz & Weatherley (1982) tested their hypothesis of root contraction and subsequent vapour gap development, under high transpiration, by squeezing and vibrating the soil in which sunflower roots were growing. They found that this mechanical treatment temporarily reduced the water status of the plant. The water status was measured in situ using a $\beta$ gauge attached to the leaf. A $\beta$ gauge is a radioactive source, positioned on one side of the leaf, with a detector, positioned on the other side of the leaf; the detector measures the $\beta$ radiation and the change in the $\beta$ radiation signal, indicates a change in the leaf water content. The reduced plant water stress was thought to result from the closing of gaps that had developed between the root and soil. Closing the gaps would reduce both (a) the velocity of water at the root surface for a given flow rate (as the surface area for flow is increased) and (b) the pressure drop across the root and soil (as the required pressure drop would be less to maintain the same flow rate).

Huck, Klepper & Taylor (1970) monitored the diurnal variations in root diameter of a cotton plant grown in a rhizotron. The shrinkage occurred in the middle of the day with a 3 h lag time between the peak radiation (or transpiration demand, they only measured radiation) and the minimum root diameter. The minimum root diameter was 60% of the maximum.

Taylor & Willatt (1983) found that soybean roots shrank to 70% of their original diameter when equilibrated in a solution with an osmotic potential of –1.34 MPa. But shrank to 97% of their original diameter when equilibrated in a solution with an osmotic potential of –0.37 MPa. They also found that soybean roots shrank 2% when in moist soil (suction less than 200 kPa) with the leaves subjected to a large evaporative demand sufficient to cause wilting after 2 h. They concluded that root shrinkage when plants are growing in moist soil will probably cause negligible increases in root-soil resistance. However they could not discount that the roots may shrink appreciably as the soil dries substantially. Hamza, Anderson & Aylmore (2007) found, using X-ray computed tomography, significant shrinkage, 72.3 and 54% for radish and lupin respectively, when roots were subjected to a NaCl solution with osmotic potential of –2.0 MPa.
Nobel & Cui (1992) found that the roots of desert succulents decreased in diameter by 15 to 19% during a four to five day period when placed on a screen 5 mm above a solution with a water potential equivalent to a suction of 10 MPa, inside a sealed chamber. The temperature of the sealed chamber was maintained at $25\pm 0.2^\circ$C in a water bath and the water potential varied by adjusting the concentration of sodium chloride in the solution. When the shrunken roots were re-hydrated by placing them in the chamber, this time with a water potential equivalent to a suction of 0.01 MPa, they returned to within 1 to 3% of their original diameter within four to five days. The recovery of the roots would facilitate water uptake when the soil is re-wet and this characteristic may be important for wheat crops that receive rainfall after dry periods.

North & Nobel (1997a) measured 24 and 34% root shrinkage after 7 and 14 days of drought respectively for a desert succulent. They found that the resulting root-soil air gaps were greatly diminished when the containers housing the soil and roots were shaken and that shaking facilitated root water uptake. The root-soil air gaps were measured by taking resin casts \textit{in situ} of the soil and roots. The resin casts were sectioned into 1 cm transverse and longitudinal sections and measurements of the air gap and root diameter made using a dissecting microscope. The resin technique was also used by North & Nobel (1997b) where they found that cactus roots shrunk 30% when subjected to drought treatment, reducing root-soil contact from 81 to 31%, thereby reducing water loss from the root to the dry soil; a desirable characteristic for desert plants.

Attempts to model root shrinkage and the resulting decrease in root-soil contact have been made (Willigen 1984; Nye 1994). The study by Nye considered roots that had forced their way through moist soil. When the soil dries, the root shrinks and the model assumes that the root retains some contact with the soil due to surface tension. He then calculated the flow of water from the radial zone, defined using eq. 2, in the bulk soil to the root endodermis. The movement of water vapour, corrected for thermal effects (Cowan & Milthorpe 1968), was calculated for flow across the air gap. Nye (1994) calculated that root shrinkage of 10% could reduce water inflow by a factor three. It is worth noting that vapour flow may be reversed, where vapour flows from the root to the surrounding atmosphere, if respiration sufficiently warms the root so that the vapour pressure at the root surface becomes greater than that in the surrounding atmosphere (J. Boyer, pers. comm.).

10
The dynamics of root shrinkage and its effect on root uptake for wheat plants requires further investigation. Although Nobel and colleagues have studied root shrinkage of cactus and other dessert succulents, it is unclear if their findings would translate across to agronomic wheat plants. However it appears that the effect of root shrinkage on the extraction of water from the soil by wheat roots can not be discounted.

1.3.5 Osmotic pressure at the root surface

Nulsen & Thurtell (1980) attributed a decrease in leaf water potential, when the osmotic potential of the soil solution was decreased, to a build up of solute just outside or just inside the cortex of the root. They used nutrient solutions with osmotic potentials of −70, −190 and −380 kPa and then leached the soil with distilled water. After leaching the soil with distilled water, the leaf water potential of the plant growing in nutrient solution of −190 kPa osmotic potential rose from −400 kPa to −250 kPa. Similarly, when the soil with the plant growing in nutrient solution of −380 kPa osmotic potential was leached with distilled water, the leaf water potential rose from −550 kPa to −300 kPa.

Stirzaker & Passioura (1996) found that the cylindrical diffusion equation, (eq. 1) solved numerically for the flow of water to an idealized plant root, approximately described the fall in leaf-xylem hydrostatic pressure as the soil dried, for tomato plants whose soil had been leached with water. Unlike Herkelrath, Miller & Gardner (1977a) and Bristow, Campbell & Calissendorff (1984), invoking an interfacial resistance between the soil and root was not required. However, when the soil was flushed with a nutrient solution with an osmotic potential of 70 kPa, the fall in hydrostatic pressure of the leaf xylem was several times faster than that in the soil. Essentially, the plants had greater difficulty in extracting water from the soil than would be expected from the sum of bulk osmotic and matric potentials. They hypothesized, like Nulsen & Thurtell (1980), that solutes accumulated either in the root or just outside it, thereby creating large osmotic pressures (an osmotic choke), which gave the appearance of an interfacial resistance.

1.3.6 Hydraulic resistance of the plant

Blizzard & Boyer (1980) measured the resistance to water transport in soybean plants grown in pots. The plant transpiration rate and the suctions of the soil, root and leaf
were measured. The plant resistance was larger than the soil resistance across the entire range of soil water availability. This paper remains unique, as it is the only study in which the water potentials of the soil, root and leaf and the plant transpiration rate were all measured directly. The difference in water potential between the leaf and root remained constant across the range of transpiration rates at 250 kPa. Similarly, the suction between the soil and root also remained constant, at a lower nominal value of 100 kPa. However, the transpiration rate decreased non-linearly as the soil dried. As a result, the resistance to water flow between the root and leaf increased at a greater rate than the resistance between the soil and root (refer to their Figures 4 and 5). This finding is important because most people have found; 1) reasonably constant resistance to water flow within the plant and 2) that the soil or soil-root interface provides the greatest resistance to water flow within the system. The increased resistance within the plant may have been due to cavitation of the xylem vessels.

1.4 General plant water uptake models

In crop simulation modelling typically the flow of water to the plant roots is described (as identified in a recent review (Green, Kirkham & Clothier 2006)) as either microscopic (single root model) or macroscopic (distributed sink).

Crop simulation models that employ root water uptake functions based on the single root model are the basis of the approach employed in the CERES-wheat (Ritchie 1985), APSIM-SWIM (Verburg, Ross & Bristow 1997), CROPGRO-soybean (Calmon et al. 1999) and FUSSIM modules (Heinen & de Willigen 1998; Heinen 2001). Macroscopic plant water uptake functions are typically an empirical function that describe water uptake in response to soil suction, soil water content or time. The mathematical analyses have been reviewed recently by Raats (2007). Others have reviewed plant water uptake functions with particular reference to crop models (Wang & Smith 2004). A comprehensive review on plant water and nutrient uptake modelling was presented by Hopmans & Bristow (2002). A more recent review (Green, Kirkham & Clothier 2006) discusses the requirement to better infer the soil water potential at the soil root boundary and its control on plant transpiration. The technology advances in instrumentation are mooted as one way of increasing our understanding of root function (Clothier & Green 1997) and thereby developing more mechanistic models.
The macroscopic flow models typically describe plant water uptake with an empirical exponential decay function (Passioura 1983). In the APSIM models that employ a so-called cascading layer (Keating et al. 2003), this function is empirically parameterized according to the root length density \( L_v \) and soil water diffusivity \( D \); where \( D \) and \( L_v \) are lumped together to form a time constant for the exponential decrease in the soil water content with time. A similar approach has been used to describe plant water uptake for various crops using only drained upper limit, lower limit and one exponential fitting parameter (Dardanelli et al. 2004).

Gardner (1991) proposed a model for plant water uptake based on a distributed sink moving downward through the soil profile. The empirical model uses two parameters: root depth and extractable water. Another empirical model was developed by observing plant water uptake in a drying soil relative to a well watered soil and observing the response in transpiration as the soil water content decreased (Sinclair 2005). The simple derivation predicted plant water uptake that was consistent with experimental data and independent of root length density, transpiration rate and soil depth.

The so-called Feddes reduction function (Feddes et al. 1976) is the basis for other empirical plant water uptake models (van Genuchten 1987; Hopmans & Guttierezrave 1988). In these models, the water uptake by roots is represented by adding a sink term to the Richards flow equation for soil water flow. The sink term comprises the reduction function where, in the original work of Feddes et al. (1976), water uptake is zero for soil suctions drier that 1500 kPa and wetter than 5 kPa. For soil suctions between 5 and 400 kPa, plant water uptake by the roots is constant at the maximum rate. Water uptake then decreases linearly with decreasing soil water content from 400 to 1500 kPa soil suction. Water uptake is evenly distributed over the rooting depth. Plant water uptake is thereby dependant on the average soil suction or osmotic pressure of the soil water, where the potential transpiration is partitioned to each soil layer and the actual uptake for each layer estimated as the product of the partitioned potential transpiration and the local reduction factor.

The Feddes reduction function is the basis for the plant water uptake routine in the popular HYDRUS-2D flow model (Simunek, Sejna & van Genuchten 1999). A two-dimensional root water uptake function based on the exponential function of Raats
(1974) was incorporated in HYDRUS-2D to simulate the root water uptake from an almond tree in two and three dimensions (Vrugt et al. 2001). The function proposed by Raats (1974) is:

\[ \beta(z) = \frac{1}{\lambda} e^{-\frac{z}{\lambda}} \]

where \( \beta(z) \) [m\(^{-1}\)] is the spatial root water uptake distribution with depth, \( z \) [m], and \( \lambda \) [m] is selected so that at depth \( \lambda \) the cumulative root water uptake, in the region 0 to \( \lambda \), is 63% of the total uptake over the whole root zone. Vrugt, Hopmans & Simunek (2001) and Vrugt et al. (2001) found that, using the optimized water flow and root uptake parameters, the simulated and measured values were in good agreement. This approach involves a high number of spatial and temporal soil water content measurements for model calibration. However, the model then predicts for the specific soil and plant conditions and is useful for investigating different irrigation and climatic scenarios. This method may also provide insight into compensatory mechanisms that the root system may use to preferentially extract water from wetter parts of the soil profile.

1.5 Hypotheses for what is limiting uptake of water by plant roots

In summary, the hypotheses for what is limiting the uptake of water by plant roots include:

- Soil limiting, where the main resistance to flow is within the soil as described by Philip (1957c), Gardner (1960) and Cowan (1965). This includes root clumping in biopores as described by Passioura (1991).
- Interfacial resistance, where the main resistance to flow is at the soil-root interface as discussed in Bristow, Campbell & Calissendorff (1984). This includes resistance within the root itself and a loss of contact between the root and soil as the soil dries, as described by Herkelrath, Miller & Gardner (1977a).
- Plant limiting, as observed in the experiment of Blizzard & Boyer (1980), where the plant resistance was larger than the soil resistance across the entire range of soil water availability.
- Build up of osmotic pressure at the root surface; as discussed in Stirzaker & Passioura (1996) where it was speculated that solutes carried convectively with the transpiration stream build up at the root surface, thereby resulting in an increase in osmotic pressure at the root surface as the transpiration rate increases and the soil water content decreases.
The aim of this thesis is to test the hypothesis that the soil is the main resistance to the extraction of water by plant roots, owing to a combination of low $D(\theta)$ and low $L_r$ at low $\theta$. This hypothesis will be tested by comparing plant water uptake laboratory experiments with a model that solves the radial diffusion equation for the flow of water to a single plant root.
Chapter 2  Method development for measurement of soil water diffusivity

2.1  Introduction

Soil water diffusivity, \( D(\theta) \) [m\(^2\) s\(^{-1}\)], is the product of the hydraulic conductivity, \( k \) [m s\(^{-1}\)], and the slope of the soil water retention curve at a particular soil volumetric water content, \( \theta \), where \( \tau \) is the soil matric suction (expressed in meters here to simplify the calculation of \( D(\theta) \), but in kPa in the rest of the thesis):

\[
D(\theta) = k(\theta) \times \frac{d\tau}{d\theta}
\]  

(3)

The relationship between the flux of water, \( F \) [m s\(^{-1}\)], \( k \) and \( D(\theta) \) is:

\[
F = D(\theta) \frac{d\theta}{dx} = k(\theta) \frac{d\tau}{dx}
\]  

(4)

For modelling plant water uptake from the soil, \( D(\theta) \) must be known in the range of \( \tau \) from 100 to 1500 kPa. It is important to note that when using \( D(\theta) \) to model flow of water through soil the gradient in \( \theta \) is implicitly used to represent a gradient of soil water suction, which is the true driving force. This means that when using \( D(\theta) \) to model flow of water through soil the modelling domain of interest must share the same \( \theta(\tau) \) relationship; i.e. the soil can’t be layered with different soil types, have appreciable hysteresis effects or be subject to thermal gradients. Under these conditions flow may not necessarily occur down gradients in \( \theta \). However, whenever the use of \( D(\theta) \) can be justified, an important advantage is that its range of variation is typically much smaller than that of \( k \). The maximum value of \( D(\theta) \) is usually of the order 10\(^{-5}\) [m\(^2\) s\(^{-1}\)] and diminishes with falling \( \theta \) to a minimum of about 10\(^{-8}\) to 10\(^{-9}\) [m\(^2\) s\(^{-1}\)]. In the same \( \theta \) range, \( k \) varies about a millionfold whereas \( D(\theta) \) varies about a ten thousandfold. Therefore small changes in \( \theta \) are likely to affect the value of \( D(\theta) \) to a much lesser extent than \( k \) (Hillel 1998).

There is little published information on soil water diffusivity of undisturbed soil in the range from 100 to 1500 kPa. The measurement of soil water diffusivity on re-packed soil is more common. The method of Rose (1968) uses evaporation under a turbulent condition, and sections the core during stage two drying, when evaporation is proportional to \( t^{\frac{1}{2}} \), to determine the water content profile. \( D(\theta) \) is then determined
using the analysis of Matano (1933). On undisturbed soil, this technique is experimentally difficult as the soil becomes very hard when air dry and difficult to section. Also, the effects of heterogeneity on water holding capacity, even at small scales, limits the reliability of the analysis.

Numerous attempts have been made to measure $D(\theta)$ using the one-step outflow method, some of these include: Crescimanno & Iovino (1995), Dam, Stricker & Droogers (1992), Doering (1965), Gardner (1956), Gupta, Farrell & Larson (1974), Hopmans, Vogel & Koblik (1992), Muazu, Skopp & Swartzendruber (1990), Passioura (1976), Valiantzas, Londra & Sassalou (2007) and Zia-ul-Haque (1990). The outflow method involves measuring the outflow of water with time from a soil sample, with a uniform initial water content, $\theta_i$, contained in a pressure cell which is subjected to a single pneumatic pressure step. The outflow is measured at one end where the cell is sealed with a membrane permeable to water but not gas.

The analysis of Passioura (1976) assumes that $\theta$ at the outflow end is reduced to the final water content, $\theta_f$, at the onset of outflow. This technique has only been used to determine $D(\theta)$ at $\tau$ wetter than 700 kPa, and the analysis assumes that the $D(\theta)$ function is monotonic and does not increase with decreasing $\theta$, as sometimes happens at very low $\theta$.

This chapter describes a simple method for measuring $D(\theta)$ on undisturbed and repacked soil, using evaporation, to permit modelling the flow of water to plant roots in the range from 100 to 1500 kPa $\tau$. The method involves continuous measurement of the evaporation with time from the top of a soil core, of known $\theta_i$, that is sealed at the base, until the evaporation becomes negligible. The $D(\theta)$ is calculated using an iterative process with two main steps; (1) the evaporation from a one-dimensional soil profile is modelled using the diffusion equation in finite difference form; a constant water content boundary condition is set at the evaporating surface, that is equal to $\theta_f$ from the experiment and (2) the $D(\theta)$ function is continually optimized with the downhill simplex optimization routine (Nelder & Mead 1965), using the least squares criterion, to minimize the error between the experiment and the model.
2.1.1 Theory

Three stages of evaporation or outflow are referred to when describing evaporation and outflow experiments. These are defined as follows (Crank 1975, p. 33): during stage one the plot relating cumulative evaporation with $t^\frac{1}{2}$ is concave up (Figure 1(b)) and the cumulative evaporation is linear with time (Figure 1(a)). During stage one the evaporation rate is a function of the boundary layer at the soil surface and the soil itself is not limiting the evaporation rate. During this period the system is not isothermal, as there is a temperature depression at the soil surface due to latent heat of evaporation.

After some time the evaporation rate decreases as the soil surface dries and starts to reduce the difference in vapour pressure across the boundary layer. Soon, $\theta_f$ is attained at $x=0$, which marks the start of stage two. The core sample behaves semi-infinitely and cumulative outflow is a linear function of $t^\frac{1}{2}$ (Figure 1(b)). During this stage the change in water content at the evaporating surface has not yet perturbed the water content at the end of the column; i.e. there is no noticeable change in $\theta$ at the non-evaporating end, $x = L$.

During stage two the evaporation rate is a function of the soil’s ability to transmit water to the evaporating surface. The linear behaviour of evaporation with $t^\frac{1}{2}$ indicates that $\theta$ at the evaporating surface is now constant (Crank 1975, p. 37). The slope of the line relating the evaporative flux with $t^\frac{1}{2}$ during stage two is the desorptivity, $D_e$. Numerous authors have derived formulas relating $D(\theta)$ to $D_e$ (Lockington 1994). Finally, stage three develops when $\theta$ at the non-evaporating end, $x = L$, starts to decrease to the final $\theta$ and the material ceases to behave semi-infinitely. Evaporation is then no longer linear with $t^\frac{1}{2}$ (Figure 1(b)).
Evaporation takes place at the top end, $x = 0$, where, for the analysis, it is assumed that the water content is reduced to $\theta_f$ at the onset of evaporation. Neglecting gravity for the short columns used in this study, the diffusion equation for one-dimensional flow of water in a stable soil of finite length is eq. 5 subject to the conditions in eq. 6.

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} \left( D(\theta) \frac{\partial \theta}{\partial x} \right)$$  \hspace{1cm} (5)

$$\begin{align*}
\theta &= \theta_i, & 0 \leq x \leq L, & t = 0 \\
\theta &= \theta_f, & x = 0, & t > 0 \\
\frac{\partial \theta}{\partial x} &= 0, & x = L, & t > 0 
\end{align*}$$  \hspace{1cm} (6)

However when the soil core behaves semi-infinitely ($x \to \infty$) and $\theta$ at the soil surface is maintained constant, the initial and boundary conditions are:

$$\begin{align*}
\theta &= \theta_i, & x \geq 0, & t = 0 \\
\theta &= \theta_f, & x = 0, & t > 0 
\end{align*}$$  \hspace{1cm} (7)

The Boltzmann transform (Boltzmann 1894)

$$\lambda = xt^{-\frac{1}{2}}$$  \hspace{1cm} (8)

was used by (Philip 1957b) to reduce eq.5 and eq.7 to:

$$-\frac{\lambda}{2} \frac{\partial \theta}{\partial \lambda} = \frac{\partial}{\partial \lambda} \left( D(\theta) \frac{\partial \theta}{\partial \lambda} \right)$$  \hspace{1cm} (9)

$$\begin{align*}
\lambda &= 0, & \theta &= \theta_i \\
\lambda &\to \infty, & \theta &= \theta_f 
\end{align*}$$  \hspace{1cm} (10)
From Philip (1957b) the solution of eq.5 subject to eq.7 is then a series solution that converges rapidly such that all but the first term can be ignored without series error. The first term is:

$$x(\theta, t) = \lambda(\theta) t^{\frac{1}{2}}$$

(11)

and the quantity of water, $W$ [m], lost by evaporation is

$$W = \int_{\theta_j}^{\theta} \int_{\theta_j}^{\theta} \lambda(\theta) d\theta$$

(12)

Importantly, eq. 12 shows that when the soil core behaves semi-infinitely and $\theta$ at the soil surface is maintained constant the flux is proportional to $t^{1/2}$.

In practice, $\theta_j$ is not attained at $x = 0$ instantly, for evaporation is at first only limited by the ability of the atmosphere to remove water from soil. So that initially, the rate of evaporation, $\partial E/\partial t$, is proportional to the difference in humidity between the soil surface, $h_s$, and the adjacent atmosphere, $h_0$. $\kappa$ is a proportionality constant that incorporates the boundary layer resistance between the evaporating soil surface and the air stream:

$$\frac{\partial E}{\partial t} = \kappa(h_s - h_0), \quad x = 0,$$

(13)

Experimentally, eq. 13 defines stage one evaporation, and in this system is typically less than 10 min duration. Thereafter, the evaporation rate is determined by the soil hydraulic properties; i.e. in eq. 13 the difference between $h_s$ and $h_0$ at the soil surface becomes very small and the flux is determined by the soil's ability to transmit water to the evaporating surface (Philip 1957a).

### 2.2 Evaporation experiment

Soil samples contained in cylinders 3.2 cm inside diameter by 6.0 cm height, either collected undisturbed from the field or repacked, were allowed to saturate from the base up and then drained to a suction of either 5 or 10 kPa, giving $\theta_i$. The base of the core, $x = L$, is then sealed and the top, $x = 0$, subjected to rapid evaporation. An air outlet was devised to supply a controlled, continuous flow of air (nominally 50 L min$^{-1}$) onto the top of the soil sample. The outlet of the air stream was positioned 20 cm directly above the soil surface. The velocity of the air stream was sufficiently fast to create a turbulent atmosphere just above the soil surface, thereby reducing the boundary layer between the air stream and the evaporating soil surface. Scotter & Raats (1969) showed that the dispersion of water vapour flux, due to pressure
fluctuations penetrating the soil surface, decreases rapidly with decreasing particle size of the medium. The fine particles of the clay-loam soil, predominantly used in this study, were likely to reduce the effect of pressure fluctuations penetrating the soil surface.

The water vapour concentration of the air was conditioned by passing the air-stream through a copper cooling coil and water trap that was kept in a temperature controlled water bath maintained between 0 and 1°C. The dew-point of the air stream was measured with a dew-point hygrometer (EG&G Model 911). The line between the copper cooling coil and the experiment was approximately 10 m long; enabling the temperature of the air-stream to adjust to ambient. A schematic of the system is shown in Figure 2.

![Figure 2 Schematic of air supply line for measuring evaporation from undisturbed soil samples, subject to high evaporative demand. The copper cooling coil was maintained between 0 and 1°C to keep the water vapour concentration of the air constant. Arrows denote direction of air stream flow and dotted lines indicate that the instrument was logged.](image)

The experimental setup is shown in Figure 3. The sample was positioned on a balance and the controlled air stream was automatically turned off at a set interval for 10 s with a solenoid and timer. During this time the balance stabilized and the weight was logged with a computer, enabling measurement of evaporation with time. For the first 15 min of evaporation, when the rate of evaporation was changing rapidly, the weight
was measured every 2 min. Thereafter, the sampling interval increased to 5 min, then 10 min and 20 min as the rate of evaporation diminished.

Figure 3 Experimental setup for measuring evaporation from undisturbed soil samples, subject to high evaporative demand. Where (a) is the controlled air outlet and (b) is the soil sample. Cumulative evaporation was measured by automatically turning off the air supply for 10 s, allowing the balance to stabilize, and the weight was then logged with the computer.

To ensure that the viscosity of the soil water remained constant, the experiment was housed in a temperature controlled cabinet, maintained at 29°C ± 0.5. This was checked by logging the temperature using copper constantan thermocouples positioned in two locations: inside the temperature controlled cabinet and at the air outlet inside the air stream. The diurnal fluctuation in ambient lab temperature was between 2 and 3°C and the temperature of the air stream was typically within 0.5°C of the ambient temperature inside the cabinet (Figure 4). To test for cooling on
expansion, the thermocouple located at the air outlet was placed 3 cm below the air outlet, in the air-stream: there was no noticeable evidence of cooling on expansion, because the air-stream temperature 3 cm below the air outlet was the same as the temperature at the air outlet.

![Graph showing temperature variations](image)

**Figure 4** Typical temperature plot from evaporation experiment. The constant temperature cabinet was maintained at 29°C ± 0.5 so that the viscosity of the soil water remained constant.

The temperature was measured at the evaporating surface (Figure 5) and typically the temperature depression due to evaporation was between 8 and 10°C. The maximum temperature depression was attained rapidly and lasted for approximately five min before the temperature started to increase back to that of the ambient temperature inside the cabinet. This pattern was independent of soil type, as the temperature depression was measured at the soil surface on sand, clay and clay-loam soil; both undisturbed from the field and repacked at various bulk densities (see Appendix 1).
The large initial temperature depression at the soil surface is due to latent heat required to change liquid water to vapour (Philip 1957a). The maximum temperature depression is steady for approximately five min and during this time it is reasonable to assume that liquid water is evaporating from the soil surface. The increase in temperature at the soil surface occurs after approximately five min and at the same time the rate of evaporation is falling as stage one ends and stage two begins (Figure 6). The reduction in water content at the soil surface, as stage one ends and stage two begins, results in the consumption of less latent heat of evaporation at the soil surface. After about five min the sensible heat supplied from the surrounds becomes greater than the latent heat required for evaporation, thereby resulting in the temperature at the evaporating surface gradually rising.

The temperature of the air stream is between 28 and 29°C and the dewpoint is between 0 and 1°C, which is a vapour pressure ranging from 0.6 to 0.7 kPa. Assuming initially that the relative humidity of the soil surface is close to 100%, at 28°C the vapour pressure at the soil surface would be 3.8 kPa. At the onset of evaporation, when the core is immediately subjected to the large evaporative demand of the air stream, the temperature falls to about 18°C, thereby reducing the saturated vapour pressure from 3.8 kPa (at 28°C) to 2.1 kPa (at 18°C). This effectively decreases the
gradient in vapour pressure, at the soil surface, about 50%, without accounting for the
decrease in vapour pressure due to the decrease in relative humidity in the soil as it
dries; where just a 1% fall, from 100 to 99%, in the relative humidity of the soil
corresponds to a suction of 1.4 MPa.

The effect of this decrease in vapour pressure gradient is evident when the rate of
evaporation is plotted with time (Figure 6): the rate of evaporation decreases
approximately 25% from when it is first measured at 150 sec to the second
measurement at 300 sec and from the first measurement at 150 sec to the fourth
measurement at 600 sec the rate of evaporation decreases approximately 50%.

Figure 6 Typical plot of cumulative evaporation and rate of evaporation for the first 2000 sec.
The rate of evaporation decreases approximately 25% from when it is first measured at 150 sec
to the second measurement at 300 sec. From the first measurement at 150 sec to the fourth
measurement at 600 sec the rate of evaporation decreases approximately 50%. The minimum
temperature depression at the soil surface is attained around 250 s.

The evaporation is measured by diverting the air stream from the soil and balance for
ten seconds and allowing the balance to stabilize, during the initial stages of
evaporation this would perturb the evaporation rate. The first measurement occurs
between 150 and 170 seconds after the onset of evaporation: therefore initially the
evaporation rate is decreased for 6-7% of the time. The measurement effect then decreases as the time after the onset of evaporation increases.

Figure 5 and Figure 6 can be used to demonstrate the veracity of the constant $\theta_f$ boundary condition at the soil surface, described in eq. 6, according to the following sequence of arguments:

1. Figure 5 shows that the temperature at $x = 0$ reaches a minimum at about 150 s after the onset of evaporation. The minimum stabilizes for a few minutes before the temperature starts to rise, after about 300 s, because less latent heat of evaporation is required at the soil surface and the inflow of sensible heat becomes greater than the loss of latent heat, thereby indicating that $\theta$ at $x = 0$ has fallen.

2. Figure 6 shows that after 300 s the evaporation rate from the soil surface has decreased at least 15 to 20%, yet after 300 s Figure 5 shows that the temperature at $x = 0$ has started to rise. If the vapour pressure at $x = 0$ remained saturated, the rate of evaporation would increase as the temperature increased. It does not; the rate of evaporation continues to decrease while the temperature at $x = 0$ continues to increase.

3. For the rate of evaporation to continue to decrease, while the temperature is rising at $x = 0$, the difference in vapour pressure across the boundary layer between the soil surface and the air-stream must also fall significantly.

4. Because the temperature is rising at $x = 0$, while the rate of evaporation is falling, the vapour pressure at $x = 0$ must be falling below the saturated vapour pressure and therefore, the relative humidity at the soil surface must also be falling.

5. Figure 6 shows that from 300 s to 500 s the rate of evaporation has fallen about 25%, while the temperature at $x = 0$ (Figure 5) has risen about 2°C. Therefore the vapour pressure at the soil surface has also decreased significantly and because the temperature at $x = 0$ has only risen 2°C, the relative humidity at the soil surface must also decrease by about 15 to 20%.
therefore the relative humidity at the soil surface after five or seven minutes is at most 90%.

The relative humidity in the pore space can be related to the soil water potential, $\psi$, using the Kelvin equation (Hillel 1998)

$$\psi = RTM_w^{-1} \rho_w \ln\left(\frac{p_w}{p_0}\right)$$

(14)

where $R$ is the universal gas constant, $T$ is the absolute temperature, $M_w$ is the molecular weight of water, $\rho_w$ is the density of liquid water, $p_w$ is the partial pressure of water vapour and $p_0$ is the saturation vapour pressure of water at the given temperature. The ratio of $p_w$ to $p_0$ is the relative humidity.

Using eq. 14 the $\psi$ at 90% RH and 28°C is 14.6 MPa. At this $\psi$ it is reasonable to assume that $\theta$ at $x = 0$ is negligibly different from $\theta_f$, and the use of the constant $\theta_f$ boundary condition at the soil surface, described in eq. 6, is justified. Evaporation from the soil surface is modelled using eq. 5 and the conditions in eq. 6, both of which assume isothermal conditions. Figure 5 shows that the temperature at the soil surface asymptotically rises back to that of ambient after about 2000 s, which equates to less than 0.5% of the total experiment time, thereby justifying the assumption of isothermal conditions.

### 2.3 Modelling evaporation

A program was written in MATLAB, modified from Campbell (1985), that simulates the evaporation from a one dimensional soil profile. Each experiment was simulated, using the experimental values of $\theta_i$, $\theta_f$, the length of the sample and an initial estimate of $D(\theta)$. The error between the experiment and simulation was minimized by adjusting $D(\theta)$ using the downhill simplex optimization routine (Nelder & Mead 1965).

The diffusion equation, eq. 5, can be solved by replacing spatial and temporal derivatives by suitable approximations and numerically solving the resulting difference equations. Instead of solving for $\theta(x,t)$ with $x$ and $t$ continuous, $\theta_{ij} = \theta(x_i,t_j)$ are solved for, where $(x_i = i \delta x)$, $(t_j = j \delta t)$, are defined on the below grid (Figure 7).
Figure 7 Grid representing discretization of spatial \((x_i = i\delta x)\) and temporal derivatives \((t_j = j\delta t)\), used to express the diffusion equation in finite difference form.

When the diffusion coefficient is constant, eq. 5 can be expressed as

\[
\frac{\partial \theta}{\partial t} = D \frac{\partial^2 \theta}{\partial x^2}
\]  

(15)

and solved in a finite difference form.

The derivatives of \(\theta\) are approximated in terms of the values of \(\theta\) at grid points. The first derivative of \(\theta\) at the grid point \((x, t) = (x_i, t_j)\) can be approximated by central difference:

\[
\frac{\partial \theta}{\partial x} \approx \frac{\Delta \theta}{\Delta x} = \frac{\theta_{i+1,j} - \theta_{i-1,j}}{2\delta x}
\]

(16)

The second derivative at the grid point \((x_i, t_j)\) can be approximated using:

\[
\frac{\partial^2 \theta}{\partial x^2} \approx \frac{\Delta (\Delta \theta)}{\Delta x} = \frac{\Delta \theta}{\Delta x}\frac{\Delta \theta}{\Delta x}
\]

(17)

The spatial derivatives are evaluated using central difference at the points \(\{x_{i+\frac{1}{2}}, x_{i-\frac{1}{2}}\}\):

\[
\frac{\partial \theta}{\partial x} \bigg|_{x_{i-\frac{1}{2}}, j} \approx \frac{\theta_{i+1,j} - \theta_{i,j}}{\delta x} \quad \frac{\partial \theta}{\partial x} \bigg|_{x_{i+\frac{1}{2}}, j} \approx \frac{\theta_{i,j} - \theta_{i-1,j}}{\delta x}
\]

\[
\frac{\partial \theta}{\partial x} \bigg|_{x_{i-\frac{1}{2}}, j} \approx \frac{\theta_{i,j} - \theta_{i-1,j}}{\delta x} \quad \frac{\partial \theta}{\partial x} \bigg|_{x_{i+\frac{1}{2}}, j} \approx \frac{\theta_{i+1,j} - \theta_{i,j}}{\delta x}
\]

(18)

Then
The concentration derivative with respect to time \( \frac{\partial \theta}{\partial t} \) at the point \((x_i, t_j)\) can be calculated using forward difference:

\[
\frac{\partial \theta}{\partial t}
= \frac{\theta_{i,j} - \theta_{i,j+1}}{t_{j+1} - t_j} = \frac{\theta_{i,j} - \theta_{i,j+1}}{\delta t}
\]

The finite difference form of eq. 15, when \( D \) is constant, is then:

\[
\frac{\theta_{i,j} - \theta_{i,j+1}}{\delta t} = D \left( \frac{\theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j}}{(\delta x)^2} \right)
\]

When \( D \) is not constant and is a function of concentration and/or time, as with soil water, eq. 5 can be expressed in finite difference form by first expanding it to give:

\[
\frac{\partial \theta}{\partial t}
= D(\theta) \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial D(\theta)}{\partial x} \frac{\partial \theta}{\partial x}
\]

To convert eq. 5 into finite difference form, the right-hand term of eq. 22 must first be expressed in finite difference form. From eq. 16 it follows that:

\[
\frac{\partial D(\theta)}{\partial x} \frac{\partial \theta}{\partial x}
= \left( \frac{D_{i+1,j} - D_{i-1,j}}{x_{i+1,j} - x_{i-1,j}} \right) \left( \frac{\theta_{i+1,j} - \theta_{i-1,j}}{2\delta x} \right)
\]

Eq. 5 in finite difference form is then:

\[
\frac{\partial \theta}{\partial t}
= \frac{\partial}{\partial x} \left( D(\theta) \frac{\partial \theta}{\partial x} \right)
\]

\[
\frac{\theta_{i,j} - \theta_{i,j+1}}{\delta t}
= D_{i,j} \left( \frac{\theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j}}{(\delta x)^2} \right) + \left( \frac{D_{i+1,j} - D_{i-1,j}}{2\delta x} \right) \left( \frac{\theta_{i+1,j} - \theta_{i-1,j}}{2\delta x} \right)
\]

\[
0 = D_{i,j} \left( \frac{\theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j}}{(\delta x)^2} \right) + \left( \frac{D_{i+1,j} - D_{i-1,j}}{2\delta x} \right) \left( \frac{\theta_{i+1,j} - \theta_{i-1,j}}{2\delta x} \right) - \left( \frac{\theta_{i,j} - \theta_{i,j+1}}{\delta t} \right)
\]

Eq. 24 is solved by taking the derivative at the three space nodes \((\theta_{i-1}, \theta_i, \theta_{i+1})\) on the \(j^{th}\) time level (eq. 25) and forming a Jacobian matrix. The system of linear equations is then solved using the Thomas algorithm (Conte & deBoor 1972).
\[ \partial \theta_{i+1} = D_{i} \left( \frac{1}{\delta x^2} \right) + \left( \frac{D_{i+1,j} - D_{i-1,j}}{4 \delta x} \right) \]
\[ \partial \theta_{i} = D_{i} \left( \frac{-2}{\delta x} \right) - \frac{1}{\delta t} \]
\[ \partial \theta_{i+1} = D_{i} \left( \frac{1}{\delta x^2} \right) + \left( \frac{D_{i+1,j} - D_{i-1,j}}{4 \delta x^2} \right) \]
\[ \partial \theta_{i} = D_{i} + \left( \frac{D_{i+1,j} - D_{i-1,j}}{4} \right) \]

(25)

The program essentially solves eq. 5 subject to the conditions in eq. 6. Where a constant concentration boundary condition is set at the evaporating surface by fixing the water content at the soil surface equal to the final water content from the experiment. A zero flux boundary condition is set at \( x = L \).

The program converges fast, with typically no more than four iterations per time step required to ensure satisfactory convergence when the mass balance error threshold is \( 1.0 \times 10^{-6} [\text{cm}^3 \text{cm}^{-3}] \) per time step. That is when the right-hand side in the final line of eq. 24 is less than \( 1.0 \times 10^{-6} \).

The model was tested by comparing, at each time step, two different calculations of the quantity of water, \( W [\text{m}] \), lost by evaporation; (1) by subtracting the integral of \( \theta \) with distance at a particular time, \( t = i \), from the initial quantity of water in the soil, \( W_0 \), (eq. 26), and (2) the integral of the flux density with time at the evaporating surface using eq. 27.

\[ W = W_0 - \left[ \int_{x=0}^{x=L} \theta(x,i) \, dx \right]_{x=i} \]  

(26)

\[ W = \left[ \int_{t=0}^{t=i} D(\theta) \frac{\partial \theta}{\partial x} \, dt \right]_{x=0} \]  

(27)

Eq. 27 is calculated assuming that the distribution of soil water content with distance is parabolic for the first three nodes (\( n(1) \), \( n(2) \) and \( n(3) \)) at the soil surface. The first derivative is then a straight line and the second derivative is constant. It is then assumed that the parabola extends across the soil surface into the atmosphere to a fictitious node (\( n(0) \)). The second derivative of soil water content with distance is then:
\[
\frac{\partial^2 \theta}{\partial x^2}_{n=2} = \frac{(n(1)-2*n(2)+n(3))}{\delta x^2}
\]

(28)

Because \(\theta(x)\) is assumed parabolic the second derivative at \(n(2)\) is equal to the second derivative at \(n(1)\) or the soil surface.

\[
\frac{(n(1)-2*n(2)+n(3))}{\delta x^2} = \frac{(n(0)-2*n(1)+n(2))}{\delta x^2}
\]

(29)

Solving eq. 29 for \(n(0)\):

\[
n(0) = 3*n(1)-3*n(2)+n(3)
\]

(30)

Eq. 27 requires the first derivative at \(n(1)\):

\[
\frac{\partial \theta}{\partial x} \bigg|_{n=1} = \frac{(n(2)-n(0))}{2\delta x}
\]

(31)

Eq. 30 is then substituted into Eq. 31:

\[
\frac{\partial \theta}{\partial x} \bigg|_{n=1} = \frac{(-3*n(1)+4*n(2)-n(3))}{2\delta x}
\]

(32)

The accuracy of the program was tested by calculating the absolute difference between eq. 26 and eq. 27 and expressing it as a percentage of the quantity of water lost as calculated from the integral of water content with distance (eq. 26). This is the flux error, and should decrease as the size of the distance step decreases (eq. 16).

Figure 8 shows that the flux error decreases when the number of distance steps increases, this is because the flux density calculated using eq. 27 is proportional to \(\theta(x, t)\) at \(x = 0\), and the accuracy at which \(\theta(x, t)\) is estimated increases with the number of distance steps. Also the accuracy of both \(\theta(x, t)\) and \(\theta(x, t)\) will increase as the number of terms in the Taylor series expansion used to form the finite difference equation of eq. 5 increases, where the error is proportional to the order of the first term neglected (Crank 1975, p. 141). In Figure 8 the flux error for 1000 distance steps is <1%; giving confidence that the algorithms used in the model are correct.

For minimizing the error between the experiment and the model, by optimizing the diffusivity function (described in the next section, 2.4.), 100 distance were used in the model; this is because there was only negligible improvement in the fit between the experiment and model, when greater than 100 distance steps were used.
Figure 8 Flux error which is the absolute difference between eq. 26 and eq. 27 expressed as a percentage of the quantity of water lost calculated from the integral of water content with distance (eq. 26) for 10, 100 and 1000 distance steps. Time step, total length, diffusivity function, initial and final water contents were the same for each run.

2.4 Optimization of soil water diffusivity function

The downhill simplex method (Nelder & Mead 1965) was used to optimize the $D(\theta)$, to minimize the error between the experimental data and the model, using the flux calculated with eq. 26. The downhill simplex method of optimization is a form of multidimensional minimization; it is used for finding the minimum of a function of more than one independent variable. The code used was based on the "amoeba" of Press et al. (1986), converted from Fortran to MATLAB by Keffer (1999).

A simplex is a geometrical figure consisting, in $N$ dimensions, of $N + 1$ vertices (or points). In one dimensional space a simplex is a line segment, in two dimensions, a triangle and in three dimensions, a tetrahedron.

In this application there were three dimensions, and the $D(\theta)$ could be approximated using either a quadratic, eq. 33, or an exponential function, eq. 34, with three independent variables ($a_0$, $a_1$ and $a_2$).

\[
D(\theta) = a_0 + a_1 \theta + a_2 \theta^2 \tag{33}
\]

\[
D(\theta) = a_0 + a_1 \exp(a_2 \theta) \tag{34}
\]
Both functions typically reached the same minimum $D$, but the exponential function increased sharply at high water content compared to the quadratic function. However, in the range 500 to 1500 kPa $\tau$ both functions gave essentially the same result, which is of major importance for modelling the flow of water to plant roots, because in this range soil water is typically limiting plant water uptake.

The parabolic $D(\theta)$ function (eq. 33) permitted an increase in $D(\theta)$ at low water content as liquid transport decreases and vapour transport increases (Rose 1968). When the exponential function, eq. 34, was used in the optimization routine, the $a_0$ parameter represented the minimum $D(\theta)$, the $a_1$ parameter was typically a very small number of the order $10^{-12}$ and the $a_2$ parameter was a relatively large number typically close to 30. The exponential function, eq. 34, was very sensitive to the $a_2$ parameter because it was located inside the exponent and when eq. 34 was used in the optimization routine, the finite difference scheme sometimes did not converge. This was probably because of the very steep $D(\theta)$ function at high water content resulting in large gradients of $D(\theta)$ with distance, thereby making eq. 24 difficult to solve.

The task of the downhill simplex was to optimize the three parameters in eq. 33 in order to minimize the error between the experimental data and the model, using the least squares criterion.

There were $n$ experimental data points, with the dependant variable, the cumulative evaporation ($e_1, e_2, \ldots, e_n$) and the independent variable, the elapsed time ($t_1, t_2, \ldots, t_n$). There were also $n$ predicted cumulative evaporation values from the model ($e'_1, e'_2, \ldots, e'_n$), which must correspond to the $t_n$ set. The sum of the squared residuals ($SS_R$) is written $\sum (e_i - e'_i)^2$. The lower the $SS_R$ the better the model fitted the data.

The frequency at which the experimental data points were measured varies: 2 min during stage one, 5 min during stage two and 10 then 20 min during stage three. The calculation of the model evaporation data points depended on the time step used. To calculate the $SS_R$ each set of $(e_n - e'_n)^2$ had to correspond to the same time, $t_n$. This was achieved by approximating the experimental data at the same time points that the
model used. The experimental data was approximated with a Piecewise Cubic Hermite Interpolating Polynomial in MATLAB at the specified time points.

Using the downhill simplex to find the minimum, a three, or \( N \), dimensional simplex with four, or \( N + 1 \), vertices was created. The vertices of the simplex were characterized by four values; the \( D(\theta) \) variables, \( a \), \( b \) and \( c \), and the response, \( SS_R \). The simplex was started by providing an initial estimate of the \( D(\theta) \) variables, \( a \), \( b \) and \( c \), which were used to generate the initial matrix of variables. If an initial variable is taken as \( P_0 \), then the matrix of initial variables is given by:

\[
P_i = P_0 + \lambda \mathbf{e}_i
\]

(35)

Where \( \lambda \) is a constant (essentially an estimate of the characteristic length scale of the problem) and the \( \mathbf{e}_i \)'s are \( N \) unit vectors. The evaporation was then modeled and the \( SS_R \) calculated for each \( N + 1 \) set of variables, and then the following matrix was generated

\[
\begin{array}{cccc}
a & b & c & SS_{R1} \\
\lambda a & b & c & SS_{R2} \\
a & \lambda b & c & SS_{R3} \\
a & b & \lambda c & SS_{R4}
\end{array}
\]

(36)

where the variables denoted with \( \lambda \) are offset from the initial guess.

The downhill simplex then took a series of steps; either reflection, expansion, or contraction, so that at each stage in the process the largest point (greatest \( SS_R \)), \( P_h \), was replaced with a new point. \( P_0, P_1, \ldots P_N \) were the \( (N + 1) \) points defining the initial simplex in \( N \) dimensional space and \( \bar{P} \) was the centroid of the points. \( SS_{Ri} \) was the sum of the squared residuals at the point \( P_i \). The high and low \( SS_R \) are denoted \( SS_{R_h} = \max(SS_{Ri}) \) and \( SS_{R_l} = \min(SS_{Ri}) \). Reflection moved the point of the simplex from \( P_h \), where \( SS_{R_h} \), through the opposite face of the simplex to a lower point, \( P^* \). The co-ordinates of \( P^* \) were defined by

\[
P^* = (1 + \alpha)\bar{P} - \alpha P_h
\]

(37)

Where \( \alpha \) is a positive constant called the reflection coefficient. If \( SS_{R^*} \) (calculated from the new, reflected variables defined at point \( P^* \) lied between \( SS_{R_h} \) and \( SS_{R_l} \) then \( P_h \) was replaced with \( P^* \) and the program was looped again to start with the new simplex.
If $SS_R^* < SS_Ri$, that is, the reflection had produced a new minimum, then $P^*$ was expanded to $P^{**}$ using

$$P^{**} = P^* + (1 - \gamma)\vec{P}$$

(38)

Where $\gamma$ is the expansion coefficient, which is $> 1$. If $SS_{R^{**}} < SS_{Ri}$, $P_h$ was replaced with $P^{**}$ and the program was looped again to start with the new simplex. However if $SS_{R^{**}} > SS_{Ri}$, then the expansion had failed and $P_h$ was replaced with $P^*$ before restarting the simplex.

If the reflection from $P$ to $P^*$ resulted in $SS_R^* > SS_Ri$, for all $i \neq h$, that is replacing $P$ with $P^*$ left $SS_R^*$ the maximum, then the new $P_h$ was defined as either the old $P_h$ or $P^*$, whichever had the lower $SS_R$. Then the simplex formed a new point, $P^{***}$, by contracting according to:

$$P^{***} = \beta P_h + (1 - \beta)\vec{P}$$

(39)

Where $\beta$ is the contraction coefficient and was set between 0 and 1. The $SS_R$ was calculated from the new $P^{***}$ and if the new $SS_{R^{**}} < SS_{Rh}$, then $P_h$ was replaced with $P^{***}$ and the simplex started again. But if $SS_{R^{**}} > SS_{Rh}$, that is the contracted point was worse than $P_h$, the contraction had failed and each $P_i$ was replaced with $(P_i + P_h)/2$ and the program looped again.

The program was ended either by reaching a minimum $SS_R$ or by exceeding the maximum number of iterations, both user defined.

The ability of the downhill simplex to optimize to a unique solution was tested by numerically generating evaporation data and starting the downhill simplex optimization routine with $D(\theta)$ parameters 75% above and below the original parameters. In both cases the optimized $D(\theta)$ parameters were the same as those used in the original simulation.

### 2.5 Summary

An experimental method has been developed to measure the soil water diffusivity on undisturbed soil. The method involved measuring the evaporation from the top of a soil core of known initial soil water content that was sealed at the base. Evaporation from the soil core was continually measured until the evaporation from the soil core became negligible.
The soil core was subjected to a turbulent air stream of constant dew-point and temperature, which ensured that the soil water content at the soil surface was quickly reduced to the final soil water content. For modelling the evaporation, a constant soil water content boundary condition at the evaporating surface was used and isothermal conditions were assumed. The veracity of the boundary condition and isothermal assumption was investigated by measuring the temperature at the soil surface.

The time taken for the soil water content at the soil surface to fall to the final water content was estimated by measuring the temperature depression at the soil surface at the onset of evaporation; which typically lasts for five min regardless of the soil texture and bulk density. After about five min, the temperature at the soil surface began to increases back to that of ambient, at which point the evaporation rate had decreased around 50%. For this to occur the vapour pressure at the soil surface must also decrease significantly. The Kelvin equation can be used to relate the relative humidity of the soil to the soil water potential, where a reduction in the relative humidity at the soil surface to 90% corresponds to a soil water potential of 14.6 MPa, which is negligibly different from the final soil water content.

The analysis to calculate the soil water diffusivity function involved an iterative process with two main steps:

(1) the evaporation from a one dimensional soil profile was modelled using the diffusion equation in finite difference form. A constant soil water content boundary condition was set at the evaporating surface that was equal to the final soil water content from the experiment.

(2) The soil water diffusivity function was continually optimized with the downhill simplex optimization routine, using the least squares criterion to minimize the error between the experiment and the model. MATLAB was used for the programming.
Chapter 3  

Measurement of soil water diffusivity on undisturbed and repacked soil

3.1  Introduction

The purpose of this chapter is to measure the soil hydraulic properties, namely the soil water diffusivity, \( D(\theta) \) [m\(^2\) s\(^{-1}\)], and the soil water retention, on clay-loam field soil (undisturbed and repacked) and repacked sand. These soils were used for plant water uptake experiments (Chapter 5) and the soil hydraulic properties were required for modelling the flow of water to plant roots for comparison of the experimental data with the model.

\( D(\theta) \) was measured using three methods (1) the evaporation method described in Chapter 2, (2) the one-step outflow method where the data is analysed using the analysis of Passioura (1976) and the modelling and optimization routine described in Chapter 2, and (3) a point measurement of \( D \) made on the undisturbed soil at 8 kPa suction using the product of the hydraulic conductivity, \( k \) [m s\(^{-1}\)], and the slope of the soil water retention, \( \frac{\delta \sigma}{\delta \theta} \) [m].

The one-step outflow experiment is isothermal and was used to test the assumption of isothermal conditions, implicit in the analysis of the evaporation method described in Chapter 2, and thereby provide validation of the evaporation method. The point measurement of \( D \), calculated from \( k \) and \( \frac{\delta \sigma}{\delta \theta} \), was used to test the evaporation method at 8 kPa suction.

3.2  Method

3.2.1  Evaporation experiment

The method described in Chapter 2 was used to measure \( D(\theta) \) on three soil cores that were collected undisturbed from the field. The undisturbed soil cores were contained in 6.0 cm length by 3.2 cm diameter cylinders and collected from 30 cm depth. The field was located in south-eastern Australia (34° 43’ S, 147° 48’ E) and the soil was a red Kandosol (Isbell 2002) of light clay texture, hereafter referred to as clay-loam.
Evaporation was measured twice from each undisturbed soil core. The only difference being the initial suction: firstly the cores were equilibrated at a suction of 5 kPa and \(D(\theta)\) measured, then the cores were equilibrated at a suction of 10 kPa and \(D(\theta)\) measured again. The experiment was repeated on the same cores at different initial suctions to test the repeatability of the method.

After these measurements were completed, two of the undisturbed soil cores were then crumbled using a mortar and pestle to enable all the soil to pass through a < 2 mm sieve. All of the soil was then repacked into the original steel core to the same bulk density (1.6 g cm\(^{-3}\)) as that of the undisturbed soil using an arbor press and torque wrench.

Soil taken from the same depth and location was also crumbled and then sieved to less than 2 mm diameter and repacked to a bulk density of 1.3 g cm\(^{-3}\). A sand from the Western Australian wheat belt was also sieved to less than 2 mm and then repacked to 1.6 g cm\(^{-3}\). The undisturbed clay-loam, the clay-loam repacked to 1.3 g cm\(^{-3}\) and the repacked sand were both later used for plant water uptake experiments, where the repacked soil was packed to the same bulk density as that used here for measurement of \(D(\theta)\); the plants would not grow in the clay-loam repacked to 1.6 g cm\(^{-3}\).

The repacked soil cores were placed on filter paper and allowed to saturate from the base up on a ceramic tension table. The cores were then drained to a suction of 10 kPa and evaporation measured from the top using the same system described in Chapter 2.

The soil water diffusivity function was then calculated for each soil core using the evaporation experiment, modelling and optimization routine described in sections 2.3 and 2.4.

### 3.2.2 Measurement of hydraulic conductivity and soil water retention

The undisturbed soil cores used for measurement of \(k\) and soil water retention at the wet end were collected from the same location as those used in the evaporation experiment described in section 3.2.1. The measurements of \(k\) and soil water retention at the wet end were used to calculate \(D(\theta)\) at 8 kPa suction using

\[
D(\theta) = k(\tau) \times \frac{\delta \tau}{\delta \theta}
\]  

(40)
where $\tau$ is soil matric suction. The soil water retention at the wet end was measured on six undisturbed replicate soil cores, using a porous ceramic tension table, at suctions of 5.0, 7.5, 10.0, 12.5 and 15.0 kPa. The soil water retention was measured using pressure plate apparatus at 100, 500, 1000 and 1500 kPa suction on disturbed samples of the sand and clay-loam soil, crushed and sieved to less than 2 mm. The soil hydraulic conductivity, $k$, was measured on five undisturbed replicate soil cores, 9.8 cm inside diameter by 7.5 cm length.

A permeameter (Figure 9) was used to measure $k$. The permeameter was designed to enable a constant suction at the base of the core, which was set by the height of the drainage tube shown in Figure 9. A low flow peristaltic pump was used to apply a constant flux to the top of the soil core where a 2 to 3 cm layer of diatomaceous earth was applied: the diatomaceous earth ensured that the flux of water was distributed uniformly across the top of the soil core.

Figure 9 shows that the layer of diatomaceous earth on top of the soil core was in hydraulic continuity with a 1.15 m long column of diatomaceous earth. A water column, shown immediately to the right of the diatomaceous earth column in Figure 9, was in hydraulic continuity with the diatomaceous earth column and indicated the height of free water inside the diatomaceous earth column. The difference in height between the top of the water column and the top of the soil core was equal to the suction at the top of the soil core.

The flux density of water through the soil core was measured by recording the change in weight with time of the water supply reservoir connected to the peristaltic pump. The flow rate of the peristaltic pump was adjusted until $\delta h = \delta L$ (see Figure 9); when unit gradient pertained (i.e. $\delta h = \delta L$) the change in weight of the water reservoir was recorded. The $k$ was then calculated, at the suction set at the base of the core, using Darcy’s Law (eq. 41) (re-arranged to solve for $k(\tau)$) where $F^*$ is the flux density of water applied to the top, $A$ is the cross sectional area of the soil core, $\tau$ is the suction set at the base of core and $\delta L$ is the length of the soil core. To maintain the soil water at a constant viscosity, the permeameter was housed in a constant temperature room set at 25°C.

$$F^* = k(\tau) \times \delta h / \delta L \times A$$

(41)
Figure 9 Schematic of permeameter used to measure hydraulic conductivity on undisturbed soil cores. $\tau$ = suction at the base of the soil core and is set by the height of the drainage tube. The flow rate of the peristaltic pump is adjusted until $\delta h = \delta L$. The change in weight of the water reservoir is recorded and used to calculate $k$ at the suction set at the base of the core.

### 3.2.3 One-step outflow experiment

The evaporation method for measuring $D(\theta)$ described in Chapter 2 was compared to $D(\theta)$ obtained from one-step outflow experiments. The one-step outflow method involves a soil core subjected to a constant pneumatic pressure difference across a
water-permeable membrane located at the soil surface. The outflow of water is then measured with time. The one-step outflow method is isothermal, and was used to test the isothermal assumption used in the evaporation method described in Chapter 2 and verify the point measurement of $D$ made at 8 kPa suction in section 3.2.2.

The outflow experiment was contained in the same temperature-controlled cabinet, maintained at 29°C ± 0.5, described in section 2.2. The soil cores were placed in a Millipore pressure cell (Cat. No. XX4204700, maximum operating pressure 700 kPa) which had been modified to minimize the dead volume between the support screen and the collecting point for the outflowing water. A VC Millipore membrane (Cat. No. VCWP 04700) with 0.1 µm pore size was used and the outflowing water channeled through a 0.7 mm ID tube and deposited into a beaker containing a layer of medical grade liquid Paraffin oil on top of water. The outlet of the tube was positioned in the layer of Paraffin oil so that the outflowing liquid water descended and remained under the oil and any gas that diffused through the membrane passed up through the oil to the surface without causing any loss of water that was already inside the beaker through evaporation. The beaker was positioned on the same balance described in section 2.2 and the weight automatically logged with a computer.

The outflow was measured three times on the same undisturbed soil core, collected from the same location and having the same dimensions as those used in the evaporation experiment described in section 3.2.1. For the first outflow experiment the soil core was initially equilibrated at a suction of 5.0 kPa, to enable comparison with the point measurement of $D$ made at 8 kPa using $k$ and $\delta \sigma \delta \theta$, and the core was then subjected to a pneumatic pressure of 100 kPa. The outflow was then measured until the change in outflow with time was negligible and the core had equilibrated at 100 kPa. Then the core was subjected to a pneumatic pressure of 700 kPa, to enable comparison with the evaporation method, and the outflow measured again until the change in outflow with time was negligible. The core was then re-wet, equilibrated at a suction of 5.0 kPa and the outflow experiment repeated at 500 kPa.

The outflow was measured from the sand described in section 3.2.1, repacked to 1.6 g cm$^{-3}$, at a pneumatic pressure of 500 kPa. The sand was equilibrated at an initial suction of 10 kPa.
3.2.4 Analysis of one-step outflow data

The one-step outflow data was analysed using two methods: (1) the same method described in Chapter 2 where the outflow is modelled, assuming a constant $\theta$ boundary condition equal to the final $\theta$ of the core, and the parameters describing the $D(\theta)$ function optimized so that the model data fits the experimental data and (2) using the analysis of Passioura (1976).

The analysis of Passioura (1976) is based on the assumption that $D$ increases monotonically with $\theta$ throughout the measurement range, and that $\delta \theta/\delta t$ is constant throughout the core and therefore only data from the third stage of outflow is used. Passioura (1976) showed good agreement (see his Fig. 6) when comparing his method to the Matano analysis using the technique of Rose (1968) and when used to calculate $D(\theta)$ from numerical simulations where the $D(\theta)$ function was already known. The analysis of Passioura (1976) is briefly outlined below:

1. Cumulative outflow is plotted with $t^{1/2}$ and the third stage identified.
2. From the third stage outflow data, $D(\theta)$ is calculated using $D(\theta) = \delta O/\delta W.L^2/t/2$. Where $O$ is the rate of outflow and $W$ is the mass of water remaining in the soil.
3. The natural log of $D(\theta)$ is plotted against mean $\theta$, $\bar{\theta}$, and the slope, $B$, of this curve calculated at $\bar{\theta} = (\theta_1 + \theta_2)/2$.
4. Approximations of small and large $\theta$ are made using $\theta = \theta_1 + \pi/2(\bar{\theta} - \theta_1)$ for small $\bar{\theta}$ and $\theta = \bar{\theta} + \delta$ for large $\bar{\theta}$, where $\delta = 0.61/B$.
5. The actual $\theta$ for $D$ is determined by plotting the small and large $\bar{\theta}$s against $\bar{\theta}$ and the region where the two lines meet is then smoothed by drawing a third line between the midpoints of the first two.

3.3 Results and discussion

3.3.1 Hydraulic conductivity and soil water retention for determining soil water diffusivity on undisturbed soil at 8 kPa suction

The soil water retention data, measured on six replicate soil cores, is shown in Figure 10. A third order polynomial function was fitted to the soil water retention data with
\( \tau [m] \) as a function of \( \theta \) (eq. 42) (\( \tau \) is expressed in meters instead of kPa to simplify the calculation of \( D(\theta) \), which is expressed in \( m^2 s^{-1} \)).

\[
\tau = -154 + -4741 + \theta - 5486 + \theta^2 + 19255 + \theta^3
\]  

(42)

The first derivative of the soil water retention curve \( (\delta \tau/\delta \theta) \) is then eq. 43.

\[
\frac{\delta \tau}{\delta \theta} = -4741 + 2 \times -5486 \times \theta + 3 \times -19255 \times \theta^2
\]  

(43)

Figure 10 Undisturbed clay-loam soil: soil water retention curve. The slope of the curve was used to calculate \( D(\theta) \) at 8 kPa suction. Error bars are standard error of the mean. Six replicates were used.

The measurements of hydraulic conductivity, obtained using the method described in section 3.2.2, are shown in Table 1. For the measurement of \( k \), each replicate soil core was initially equilibrated at 1 kPa suction and \( k \) then measured. Then the core was equilibrated at 8 kPa suction and \( k \) measured again. The \( D(\theta) \) at 8 kPa suction was then calculated, using eq. 44, for each soil core replicate used to measure \( k \) (Table 1). The same value of \( \delta \tau/\delta \theta \) derived from Figure 10 was used for each calculation of \( D(\theta) \) shown in Table 1.

\[
D(\theta) = k(\tau) \times \frac{\delta \tau}{\delta \theta}
\]  

(44)
Table 1 Undisturbed clay-loam soil: soil hydraulic conductivity ($k$) measured at 1 and 8 kPa suction and soil water diffusivity ($D$) calculated from product of $k$, at 8 kPa, and slope of soil water retention curve in Figure 10.

<table>
<thead>
<tr>
<th>Replicate</th>
<th>$k$ [m s$^{-1}$]</th>
<th>$k$ [m s$^{-1}$]</th>
<th>$D$ [m$^2$ s$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 kPa</td>
<td>8 kPa</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>42x10$^{-9}$</td>
<td>3.6x10$^{-9}$</td>
<td>83x10$^{-9}$</td>
</tr>
<tr>
<td>2</td>
<td>76x10$^{-9}$</td>
<td>4.1x10$^{-9}$</td>
<td>94x10$^{-9}$</td>
</tr>
<tr>
<td>3</td>
<td>67x10$^{-9}$</td>
<td>4.4x10$^{-9}$</td>
<td>100x10$^{-9}$</td>
</tr>
<tr>
<td>4</td>
<td>58x10$^{-9}$</td>
<td>6.1x10$^{-9}$</td>
<td>140x10$^{-9}$</td>
</tr>
<tr>
<td>5</td>
<td>63x10$^{-9}$</td>
<td>6.4x10$^{-9}$</td>
<td>150x10$^{-9}$</td>
</tr>
<tr>
<td>Mean</td>
<td>61x10$^{-9}$</td>
<td>4.9x10$^{-9}$</td>
<td>110x10$^{-9}$</td>
</tr>
<tr>
<td>St dev</td>
<td>13x10$^{-9}$</td>
<td>1.2x10$^{-9}$</td>
<td>28x10$^{-9}$</td>
</tr>
</tbody>
</table>

3.3.2 Evaporation method for measurement of soil water diffusivity on undisturbed soil

Figure 11(1a) and (1c), Figure 12(2a) and (2c) and Figure 13(3a) and (3c) show evaporation measured from replicates one, two and three respectively, of the undisturbed soil cores. The evaporation experiment was repeated twice for each undisturbed soil core, where the experiments denoted “a” were initially equilibrated at 5 kPa suction and the experiments denoted “c” were initially equilibrated at 10 kPa suction.

Figure 11(1a) and (1c), Figure 12(2a) and (2c) and Figure 13(3a) and (3c) also show the real error between the experimental data and the model (real error = exp. - model) when using a quadratic (eq. 33) and exponential (eq. 34) $D(\theta)$. In most cases the real error of the exponential $D(\theta)$ and quadratic $D(\theta)$ models track each other and decrease with time. However, two points are clear from Figure 11(1a) and (1c), Figure 12(2a) and (2c) and Figure 13(3a) and (3c); (1) that the error is greatest early and (2) at approximately 10$^5$ s, the mean $\theta$ in the core is close to $\theta$ at 1500 kPa suction (core volume is 47.4 cm$^3$, $\theta$ at 1500 kPa suction is 0.14). In order to obtain a closer approximation of $D(\theta)$ in the range from 100 to 1500 kPa, an attempt was made to optimize $D(\theta)$ using only the evaporation data for the first 10$^5$ s: when compared to $D(\theta)$ optimized over the total time, the fit was slightly improved for the first 10$^5$ s; however the error was far greater over the total time, particularly for the quadratic $D(\theta)$ (see Appendix 2).
Figure 11(1b) and (1d), Figure 12(2b) and (2d) and Figure 13(3b) and (3d) show: (1) the optimized \( D(\theta) \) functions used to simulate the experimental data in each corresponding plot, (2) the point measurement of \( D \) at 8 kPa suction calculated from measurements of \( k \) and \( \delta \tau / \delta \theta \) and (3) the soil water retention data from 5 to 1500 kPa, with dashed lines denoting the range from 100 to 1500 kPa, which is of interest for modelling the flow of water to the plant roots. Figure 11(1d), Figure 12(2d) and Figure 13(3d) show that the optimized \( D(\theta) \) functions from the evaporation experiments, initially equilibrated at 10 kPa suction, start at a wetter \( \theta \) than the point measurement of \( D \) made at 8 kPa suction: this is attributed to the variation in soil water retention between the samples used for the evaporation experiment and those used for the point measurement of \( D \).

The diffusivity plot of each “b” and “d” pair is different for each of the three soil cores; however the plot of each “b” and “d” pair is similar in shape, despite equilibrating at two different suctions. In the range from 100 to 1500 kPa, the \( D(\theta) \) ranged from \( 8 \times 10^{-9} \) to \( 30 \times 10^{-9} \) [m\(^2\) s\(^{-1}\)].

The exponential \( D(\theta) \) functions in Figure 11(1b) and (1d) and Figure 12(2b) and (2d) are close to the point measurement of \( D \), calculated from \( k \) and \( \delta \tau / \delta \theta \). However Figure 11(1b) and (1d), Figure 12(2b) and (2d) show that \( D \) calculated from \( k \) is always greater than the quadratic \( D(\theta) \) function; in Figure 11(1b) and (1d) the difference is a factor three to four, in Figure 12(2b) and (2d) the difference is a factor two and in Figure 13(3b) the difference is a factor two and in Figure 13(3d) the difference is a factor five to six. The exponential \( D(\theta) \) function in Figure 13(3d) slightly decreases with increasing \( \theta \), however in the range from 100 to 1500 kPa suction the quadratic and exponential functions were in general agreement.

The discrepancy between the quadratic \( D(\theta) \) functions and \( D \) calculated from \( k \) and \( \delta \tau / \delta \theta \) is possibly due to the fact that after \( 10^4 \) seconds, about 1% of the total time, approximately one third of the total evaporation has already occurred and the mean \( \theta \) in the core is 0.2 and the mean suction is 100 kPa. Also Figure 5 shows that after \( 10^4 \) seconds the temperature at the soil surface is close to ambient: from this we can deduce that after \( 10^4 \) seconds the lower mean \( \theta \) throughout the core would make the
system almost isothermal because the latent heat requirement would be smaller and therefore the temperature depression due to evaporation would be less.

It is also worth noting that diverting the air stream to enable the balance to stabilize so the weight can be recorded is not simulated by the model. This reduces the evaporative demand for 10 seconds and the effect is greatest early on when the evaporation rate is greatest, and the proportion of time when the air-stream is diverted to when it is not, is greatest. Clearly the evaporation method is not sufficiently sensitive to give reliable measurements of $D(\theta)$ at the wet end. However the region wetter than 100 kPa suction, except in coarse sand, is irrelevant to the task of modelling water flow to plant roots.
Figure 11 Undisturbed clay-loam soil, soil core replicate one: (1a) and (1c) show experimental data and model error when using quadratic and exponential $D(\theta)$. Evaporation was measured twice on the soil core at $\theta$ corresponding to 5 kPa and 10 kPa $\tau$, as indicated on plots. (1b) and (1d) show the optimized $D(\theta)$ functions used to model each corresponding evaporation experiment shown in (1a) and (1c) respectively. Mean $D(\theta)$ at 8 kPa calculated from measurements of $k$ and soil water retention, five and six replicate soil cores respectively, shown as a single point in (1b) and (1d), error bar is standard deviation from the mean. The soil water retention is shown in (1b) and (1d) with dashed lines to denote the region of interest for modelling the flow of water to plant roots.
Figure 12 Undisturbed clay-loam soil, soil core replicate two: (2a) and (2c) show experimental data and model error when using quadratic and exponential $D(\theta)$. Evaporation was measured twice on the soil core at $\theta$ corresponding to 5 kPa and 10 kPa, as indicated on plots. (2b) and (2d) show the optimized $D(\theta)$ functions used to model each corresponding evaporation experiment shown in (2a) and (2c) respectively. Mean $D(\theta)$ at 8 kPa calculated from measurements of $k$ and soil water retention, five and six replicate soil cores respectively, shown as a single point in (2b) and (2d), error bar is standard deviation from the mean. The soil water retention is shown in (2b) and (2d) with dashed lines to denote the region of interest for modelling the flow of water to plant roots.
Figure 13 Undisturbed clay-loam soil, soil core replicate three: (3a) and (3c) show experimental data and model error when using quadratic and exponential $D(\theta)$. Evaporation was measured twice on the soil core at $\theta$ corresponding to 5 kPa and 10 kPa $\tau$, as indicated on plots. (3b) and (3d) show the optimized $D(\theta)$ functions used to model each corresponding evaporation experiment shown in (3a) and (3c) respectively. Mean $D(\theta)$ at 8 kPa calculated from measurements of $k$ and soil water retention, five and six replicate soil cores respectively, shown as a single point in (3b) and (3d), error bar is standard deviation from the mean. The soil water retention is shown in (3b) and (3d) with dashed lines to denote the region of interest for modelling the flow of water to plant roots.
3.3.3 Evaporation method for measurement of soil water diffusivity on repacked clay-loam

Figure 14(1a) and (2a) show the measured evaporation from two replicates of the undisturbed soil that was sieved to less than 2 mm and repacked to the field bulk density of 1.6 g cm$^{-3}$. Figure 14(1a) and (2a) also show the real error (real error = exp. - model) between the experimental data and the model when using an optimized quadratic (eq. 33) and exponential (eq. 34) $D(\theta)$, where the error is greatest at the start, for both the exponential and quadratic $D(\theta)$, of the experiment and reduces to an almost constant value of about ±0.1 g after about 1x10$^6$ s.

Figure 14(1b) and (2b) show: (1) the optimized quadratic (eq. 33) and exponential (eq. 34) $D(\theta)$ used to model the corresponding evaporation in Figure 14(1a) and (2a) and (2) the soil water retention in the range from 10 to 1500 kPa, with dashed lines denoting the range from 100 to 1500 kPa.

The soil water retention curve in the right-hand plots in Figure 14 shows that in the region 100 to 1500 kPa, the diffusivity ranged from 6x10$^{-9}$ to 40x10$^{-9}$ [m$^2$ s$^{-1}$], which is similar to that of the undisturbed clay-loam measured in section 3.3.2.
Figure 14 Repacked clay-loam field soil, crushed and sieved to less than 2 mm and repacked to 1.6 g cm$^{-3}$: (1a) and (2a) show experimental data of evaporation with time and model error when using quadratic and exponential $D(\theta)$. (1b) and (2b) show quadratic and exponential $D(\theta)$ used to model corresponding evaporation data shown in (1a) and (2a). The soil water retention is also shown in (1b) and (2b) and dashed lines used to indicate region of interest for modelling flow of water to plant roots.

Figure 15(1a), (2a) and (3a) show the measured evaporation from three replicates of the undisturbed soil that was sieved to less than 2 mm and repacked to a bulk density of 1.3 g cm$^{-3}$. Figure 15(1a), (2a) and (3a) also show the real error (real error = exp. - model) between the experimental data and the model when using an optimized quadratic (eq. 33) and exponential (eq. 34) $D(\theta)$, where the error is greatest at the
start, for both the exponential and quadratic $D(\theta)$, of the experiment and reduces to an almost constant value of about $\pm 0.1$ g after about $2 \times 10^5$ s.

Figure 15(1b), (2b) and (3b) show; (1) the optimized quadratic (eq. 33) and exponential (eq. 34) $D(\theta)$ used to model the corresponding evaporation in Figure 15(1a), (2a) and (3a) and (2) the soil water retention in the range from 10 to 1500 kPa, with dashed lines denoting the range from 100 to 1500 kPa.

Figure 15 shows the $D(\theta)$ measured on the soil repacked to 1.3 g cm$^{-3}$, in the region 100 to 1500 kPa varied from $10 \times 10^{-9}$ to $50 \times 10^{-9}$; the minimum is close to those shown for the undisturbed soil in Figures 11, 12 and 13.
Figure 15 Repacked clay-loam field soil, sieved to less than 2 mm and repacked to 1.3 g cm$^{-3}$: (1a), (2a) and (3a) show experimental data of evaporation with time and model error when modelled using quadratic and exponential $D(\theta)$. (1b), (2b) and (3b) show $D(\theta)$ used to model corresponding evaporation data shown in (1a), (2a) and (3a). The soil water retention is also
shown in (1b), (2b) and (3b) and dashed lines are used to indicate region of interest for modelling flow of water to plant roots.

### 3.3.4 Evaporation method for measurement of soil water diffusivity on repacked sand

Figure 16(1a), (2a) and (3a) show the measured evaporation from three replicates of the sand that was sieved to less than 2 mm and repacked to a bulk density of 1.6 g cm$^{-3}$. Figure 16(1a), (2a) and (3a) also show the real error (real error = exp. - model) between the experimental data and the model when using an optimized quadratic (eq. 33) and exponential (eq. 34) $D(\theta)$.

Figure 16(1b), (2b) and (3b) show; (1) the optimized quadratic (eq. 33) and exponential (eq. 34) $D(\theta)$ used to model the corresponding evaporation in Figure 16(1a), (2a) and (3a) and (2) the soil water retention in the range from 10 to 1500 kPa, with dashed lines denoting the range from 100 to 1500 kPa.

All three quadratic $D(\theta)$ functions in Figure 16(1b), (2b) and (3b) show rising $D(\theta)$ with decreasing $\theta$, thereby suggesting a significant contribution from vapour flow that is proximal to the contribution from liquid flow. The small contribution from the liquid phase can be attributed to the greater proportion of large diameter pores in the sand reducing the hydraulic conductivity relative to the undisturbed and repacked clay-loam soil used in Figures 11, 12, 13, 14 and 15.

The exponential $D(\theta)$ function in Figure 16(1b) reaches a maximum value of 40x10$^{-9}$ [m$^2$ s$^{-1}$] at the lowest $\theta$, a suction far drier than 1500 kPa, before decreasing with increasing $\theta$ to a constant $D$ of 6x10$^{-9}$ [m$^2$ s$^{-1}$]. This minimum is slightly less than the other two optimized exponential $D(\theta)$ functions, which resulted in constant values of $D$ of 8x10$^{-9}$ and 8x10$^{-9}$ [m$^2$ s$^{-1}$], shown in Figure 16(2b) and (3b). Figure 16(2a) and (3a) show that the error between the experiment and model for the exponential $D(\theta)$ was appreciably greater than that of the error for the exponential $D(\theta)$ in Figure 16(1a).

In the range from 100 to 1500 kPa suction $D(\theta)$, in Figure 16, ranged from 8x10$^{-9}$, for the optimized exponential function in the Figure 16(2b) and (3b), to 0.1x10$^{-9}$ [m$^2$ s$^{-1}$] for the minimum of the quadratic function in the Figure 16(1b).
The contrasting shapes of the quadratic and exponential $D(\theta)$, in Figure 16(1b), (2b) and (3b), suggest that if $D(\theta)$ is sharply quadratic around an approximately central minimum, then an exponential function would not be able to represent it. The large errors in two of the three replicates are consistent with the exponential function being inappropriate for representing $D(\theta)$ in the sand.
Figure 16 Sand repacked to 1.6 g cm$^{-3}$: (1a), (2a) and (3a) show experimental data of evaporation with time and model error when modelled using quadratic and exponential $D(\theta)$. (1b), (2b) and (3b) show quadratic and exponential $D(\theta)$ used to model corresponding evaporation shown in (1a), (2a) and (3a). The soil water retention is also shown in (1b), (2b) and (3b) and dashed lines are used to indicate region of interest for modelling flow of water to plant roots.
3.3.5 Outflow method for measurement of soil water diffusivity on undisturbed soil

Figure 17(a), (c) and (e) show data from outflow experiments (section 3.2.3) at 100, 500 and 700 kPa, where the initial suction for these cores was 5, 5 and 100 kPa respectively. The real error (real error = exp. - model) from the optimized quadratic and exponential $D(\theta)$ functions is also shown in Figure 17(a), (c) and (e). For the 100 and 500 kPa outflow experiments, the error is greatest at the start, for both the exponential and quadratic $D(\theta)$, and reduces to an almost constant value of about ±0.1 g after about $1 \times 10^5$ s. The error for the 700 kPa outflow experiment is less than ±0.1 g for the entire experiment for both the exponential and quadratic $D(\theta)$.

Figure 17(b), (d) and (f) show the optimized quadratic and exponential $D(\theta)$ functions used to model the outflow in each corresponding plot in Figure 17(a), (c) and (e). Figure 17(b), (d) and (f) also shows $D(\theta)$ calculated using the analysis of Passioura (1976). The point measurement of $D$, calculated from $k$ and $\delta\tau/\delta\theta$ in section 3.2.2, is shown in Figure 17(b) and (d). However the point measurement of $D$, measured at 8 kPa, was omitted from Figure 17(f) where the initial suction for the outflow experiment was 100 kPa.

The point measurement of $D$ made at 8 kPa suction, in Figure 17(b) and (d), was made at a greater $\theta$ than the $D(\theta)$ calculated from the outflow data, despite the initial suction of 5 kPa for the outflow experiments at 100 and 500 kPa. The discrepancy between the $\theta$ of the $D(\theta)$ from the outflow and the $\theta$ of the $D$ from the point measurement, was possibly due to the soil core that was used for the outflow experiment having a different water retention than the samples used for the point measurement of $D$; this was because the soil is undisturbed and although sampled from the same location in the field, subject to spatial variability. However, ignoring the variation in $\theta$ at high suction, the magnitude of the point measurement of $D$ at 8 kPa suction was close to the $D(\theta)$ calculated using the analysis of Passioura (1976) and the exponential $D(\theta)$. The maximum value of the quadratic $D(\theta)$ is a factor two and a factor one and a half less than the point measurement of $D$ for the outflow at 100 and 500 kPa respectively.
The $D(\theta)$ calculated using the analysis of Passioura (1976) was initially steep before reaching a constant value in all cases. The constant values were $20\times10^{-9}$, $10\times10^{-9}$ and $4\times10^{-9}$ [m$^2$ s$^{-1}$] for the 100, 500 and 700 kPa outflow experiments respectively. The shape of these curves can be attributed to the calculation of $D(\theta)$ in the analysis of Passioura (1976) depending on what is effectively the second derivative of the mean $\theta$ within the soil core with time. The $D(\theta)$ functions calculated using the optimization analysis were largely similar to the $D(\theta)$ calculated using the analysis of Passioura (1976). However, that the quadratic $D(\theta)$ functions, from the 500 and 100 kPa outflow experiments, reach a lower $D$ than the exponential and fell sharply at low $\theta$ (500 kPa more so), may be due to the quadratic function being inappropriate for representing $D(\theta)$ in this range.

The calculations of $D(\theta)$ shown in Figure 17(f), from the 700 kPa outflow experiment, range from $2\times10^{-9}$ to $7\times10^{-9}$ [m$^2$ s$^{-1}$]. The optimized exponential $D(\theta)$ was constant at $3\times10^{-9}$ [m$^2$ s$^{-1}$] and the optimized quadratic $D(\theta)$ actually decreases with increasing $\theta$, which at a suction of 500 kPa is not plausible. However the measurements from the outflow at 700 kPa vary over a small range and thereby provide justification for the use of a constant $D$ when modelling the flow of water to plant roots.

The plots of $D(\theta)$ show that for the purpose of determining the $D(\theta)$ for modelling the flow of water to the plant roots, outflow measurements at 500 and 700 kPa, using the analysis of Passioura (1976), result in a constant value of $D$ in the suction range where the soil is likely to be limiting plant water uptake.
Figure 17 Undisturbed clay-loam soil: (a), (c) and (e) show outflow experiments at 100, 500 and 700 kPa respectively and model error using quadratic and exponential $D(\theta)$. (b), (d) and (f) show quadratic and exponential $D(\theta)$ functions used to model corresponding experimental data in (a), (c) and (e) and $D(\theta)$ calculated using the analysis of Passioura (1976). (b) and (d) show $D$ calculated from $k$ and $\delta\tau/\delta\theta$.  

(a) (b) (c) (d) (e) (f)
Figure 18(a) shows the outflow at 500 kPa from sand re-packed to 1.6 g cm$^{-3}$, which was initially equilibrated at 10 kPa suction. Figure 18(a) also shows the error from the optimization for quadratic and exponential $D(\theta)$, where the error for both the quadratic and exponential $D(\theta)$ is less than $\pm 0.1$ g for the entire experiment.

Figure 18(b) shows; (1) the quadratic and exponential $D(\theta)$, used to model the outflow in the Figure 18(a), and $D(\theta)$ calculated using the analysis of Passioura (1976) and (2) the soil water retention in the range from 10 to 1500 kPa, with dashed lines denoting the range from 100 to 1500 kPa.

The $D(\theta)$ calculated using the analysis of Passioura (1976), in Figure 18(b), reaches a constant minimum of $7\times10^{-9}$ [m$^2$ s$^{-1}$] and the exponential $D(\theta)$ monotonically decreases to a minimum $D$ of $5\times10^{-9}$ [m$^2$ s$^{-1}$]. The quadratic $D(\theta)$, in Figure 18(b), reaches a minimum of $2\times10^{-9}$ [m$^2$ s$^{-1}$] which is a factor two to three less than the minimum of the exponential $D(\theta)$ and $D(\theta)$ from the analysis of Passioura (1976).

The minimum values of $D$ from the exponential and the analysis of Passioura (1976), in Figure 18(b), are close to the minimum values of the exponential $D(\theta)$ shown in Figure 16(1a), (2a) and (3a) of $6\times10^{-9}$, $8\times10^{-9}$ and $8\times10^{-9}$ [m$^2$ s$^{-1}$] respectively, where the evaporation method was used.

Figure 18 Sand repacked to 1.6 g cm$^{-3}$: (a) shows outflow experiment at 500 kPa and model error using quadratic and exponential $D(\theta)$. (b) shows quadratic and exponential $D(\theta)$ functions used to model corresponding experimental data in (a) and $D(\theta)$ calculated using the analysis of Passioura.
(1976). The soil water retention is also shown in (b) and dashed lines are used to indicate region of interest for modelling flow of water to plant roots.

### 3.4 Summary and discussion

The $D(\theta)$ and soil water retention were measured on undisturbed and repacked, replicate soil cores collected from a field in south-eastern Australia and repacked sand collected from Western Australia. Three methods were used to measure $D(\theta)$; (1) the evaporation method described in Chapter 2, (2) the one-step outflow method where the data is analysed using the analysis of Passioura (1976) and the modelling and optimization routine described in Chapter 2, and (3) a point measurement of $D$ made on the undisturbed soil at 8 kPa suction using the product of the hydraulic conductivity, $k$, and the slope of the soil water retention, $\partial \tau / \partial \theta$.

The $D(\theta)$ data is summarized in Table 2, where the minimum and maximum values, in the range from 100 to 1500 kPa suction, for each soil type for the evaporation and outflow (where measured) experiments are shown.

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>evaporation max</th>
<th>outflow max</th>
<th>evaporation min</th>
<th>outflow min</th>
</tr>
</thead>
<tbody>
<tr>
<td>undisturbed clay-loam</td>
<td>30x10^{-9}</td>
<td>20x10^{-9}</td>
<td>8x10^{-9}</td>
<td>2x10^{-9}</td>
</tr>
<tr>
<td>repacked clay-loam 1.6 g cm^{-3}</td>
<td>40x10^{-9}</td>
<td>6x10^{-9}</td>
<td>4x10^{-9}</td>
<td>1x10^{-9}</td>
</tr>
<tr>
<td>repacked clay-loam 1.3 g cm^{-3}</td>
<td>50x10^{-9}</td>
<td>10x10^{-9}</td>
<td>8x10^{-9}</td>
<td>7x10^{-9}</td>
</tr>
<tr>
<td>sand</td>
<td>8x10^{-9}</td>
<td>3x10^{-9}</td>
<td>0.1x10^{-9}</td>
<td>7x10^{-9}</td>
</tr>
</tbody>
</table>

### 3.4.1 Undisturbed clay-loam

Figure 19 compares all of the $D(\theta)$ measurements made on the undisturbed clay-loam soil. From this graph it is clear that four of the exponential $D(\theta)$ functions from the evaporation experiment were close to the point measurement of $D$ calculated from the product of $k$ and $\partial \tau / \partial \theta$ at 8 kPa suction. However two of the exponential $D(\theta)$ functions and all of the quadratic $D(\theta)$ functions from the evaporation experiment underestimated the point measurement of $D$. The discrepancy between the quadratic $D(\theta)$ functions and the point measurement of $D$, calculated from $k$ and $\partial \tau / \partial \theta$, may be due to the fact that after $10^4$ seconds, about 1% of the total time, approximately one
third of the total evaporation had already occurred, the mean $\theta$ in the core was 0.2 and the mean suction was 100 kPa, well past 8 kPa. Also, the temperature depression at the soil surface reached a minimum after about 300 s, when the mean suction in the core was close to 8 kPa, and the soil surface temperature was close to ambient after about $10^4$ s: given that the optimization analysis assumes isothermal conditions, estimates of $D(\theta)$ at the wet end may be unreliable.

Figure 19 Undisturbed clay-loam soil: collation of $D(\theta)$ measurements made using four techniques; (1) evaporation experiments and optimization analyses using quadratic and exponential $D(\theta)$, (2) outflow experiments at 100, 500 and 700 kPa and optimization analyses using quadratic and exponential $D(\theta)$, (3) outflow experiments at 100, 500 and 700 kPa and Passioura (1976) analyses and (4) point measurement of $D$ from product of $k$ and $\delta\tau/\delta\theta$. Bracketed numbers in legend denote number of replicates shown in the plot.

The 700 kPa outflow experiment was initially equilibrated at 100 kPa and therefore can not be compared to the point measurement of $D$ made at 8 kPa suction, however
three points are apparent when the point measurement of $D$ is compared to $D(\theta)$ derived from the 100 and 500 kPa outflow experiments (these are more clearly seen in Figure 17 than Figure 19); (1) all of the calculations of $D(\theta)$ derived from the 100 and 500 kPa outflow experiments start at a lower $\theta$ than the point measurement of $D$, where the point measurement of $D$ was made at 8 kPa suction and the soil core used in the 100 and 500 kPa outflow experiments was initially equilibrated at 5 kPa suction. This discrepancy is possibly due to the soil core that was used in the outflow experiments having a different soil water retention than those used in the point measurement of $D$, despite sampling all of the cores from the same location. (2) Given the proviso of the different initial $\theta$s for the outflow and the point measurement of $D$, $D(\theta)$ calculated from the analysis of Passioura (1976) and the exponential $D(\theta)$ functions from the 100 and 500 kPa outflow experiments reach approximately the same value as the point measurement of $D$. (3) The quadratic $D(\theta)$ functions from the 100 and 500 kPa outflow experiments underestimate the point measurement of $D$.

In the range from 100 to 1500 kPa suction, which is of interest for modelling plant water uptake, Figure 19 shows that all of the exponential $D(\theta)$ functions from the evaporation experiments converge on a minimum value of approximately $10\times10^{-9}$ [m$^2$ s$^{-1}$], while the quadratic $D(\theta)$ functions vary more than the exponential $D(\theta)$ functions. However at 1500 kPa suction, all of the quadratic $D(\theta)$ functions are close to $10\times10^{-9}$ [m$^2$ s$^{-1}$]. This agrees with the $D(\theta)$ calculated from the analysis of Passioura (1976) on the 500 kPa outflow data, giving confidence in the evaporation method.

In contrast to the analysis of Passioura (1976), the quadratic and exponential $D(\theta)$ functions, calculated with the optimization analysis from the 500 kPa outflow data, do not reach a constant minimum, although they are similar to each other (Figure 17). These functions differ in shape when compared to the $D(\theta)$ calculated using the analysis of Passioura (1976), yet both methods cover about the same range of $D$. That the quadratic $D(\theta)$ function, from the 500 kPa outflow experiment, reached a lower $D$ than the exponential and fell sharply at low $\theta$, may be due to the quadratic function being inappropriate for representing $D(\theta)$ in this range. The $D(\theta)$ functions calculated from the 700 kPa outflow experiment varied little with $\theta$ (as the soil core was initially equilibrated at 100 kPa suction) ranging from $1\times10^{-9}$ to $7\times10^{-9}$ [m$^2$ s$^{-1}$], and were less
than all of the $D(\theta)$ functions calculated from the evaporation experiment; thereby providing a minimum $D$ for modelling the flow of water to the plant roots.

The $D(\theta)$, calculated using the analysis of Passioura (1976) on the 100 kPa outflow experiment (Figure 17), reached a constant minimum value of $20 \times 10^{-9}$ [m$^2$ s$^{-1}$], close to the approximate minimum of the exponential $D(\theta)$ functions from the evaporation experiment. The dry end of the quadratic and exponential $D(\theta)$ functions, calculated using the optimization analysis from the 100 kPa outflow experiment, were also close to those calculated from the evaporation experiment. However, similar to the optimization analysis on the 500 kPa outflow data, the exponential $D(\theta)$ function appears to be approaching a minimum but the quadratic $D(\theta)$ function is still decreasing with $\theta$, which may be due to the quadratic function being inappropriate for representing $D(\theta)$ in this range.

Figure 19 shows that the bulk of the $D(\theta)$ measurements, in the suction range from 100 to 1500 kPa, obtain a minimum of approximately $10 \times 10^{-9}$ [m$^2$ s$^{-1}$]: this is in contrast to the work of Rose (1968), which showed that for most of the soils he used, $D(\theta)$ obtained a minimum value of approximately $1 \times 10^{-9}$ [m$^2$ s$^{-1}$] at a suction of approximately 1500 kPa. The material with the highest minimum $D(\theta)$ (approximately $4 \times 10^{-9}$ [m$^2$ s$^{-1}$]), in the study of Rose (1968), was described as subsoil clay; it is possible that soil with appreciable clay content, like the undisturbed clay-loam soil used in this study, has a greater minimum $D(\theta)$.

While there are measurements of $D(\theta)$ made on undisturbed soil found in the literature, these are predominantly made at suctions wetter than 100 kPa: no explicit measurements of $D(\theta)$ were found in the literature in the suction range from 100 to 1500 kPa, which is of interest for modelling the flow of water to plant roots. This study presents both: (1) a novel method, the evaporation method, for measuring $D(\theta)$ in the suction range from 100 to 1500 kPa and (2) novel measurements of $D(\theta)$ on undisturbed soil in the suction range from 100 to 1500 kPa.

### 3.4.2 Repacked clay-loam

Table 2 shows that when the undisturbed clay-loam soil was sieved to less than 2 mm and repacked to the same bulk density of 1.6 g cm$^{-3}$, the $D(\theta)$ ranged from $40 \times 10^{-9}$ to
6x10⁻⁹ [m² s⁻¹] in the range from 100 to 1500 kPa suction. The undisturbed clay-loam soil was also repacked to 1.3 g cm⁻³, and in the region from 100 to 1500 kPa, $D(\theta)$ varied from 50x10⁻⁹ to 10x10⁻⁹ [m² s⁻¹]. These ranges of $D(\theta)$ are similar to that of the undisturbed clay-loam soil, which in the range from 100 to 1500 kPa suction, varied from 30x10⁻⁸ to 2x10⁻⁹ [m² s⁻¹].

**3.4.3 Repacked sand**

In the range from 100 to 1500 kPa suction, the repacked sand exhibited close agreement between the three exponential $D(\theta)$ functions from the evaporation experiments, which reached constant values of 6x10⁻⁹, 8x10⁻⁹ and 8x10⁻⁹ [m² s⁻¹] (Figure 20). In contrast, the three quadratic $D(\theta)$ functions from the evaporation experiments varied with $\theta$, in the range from 100 to 1500 kPa suction, and reached minimum values about an order of magnitude lower than the exponential $D(\theta)$ functions of 0.1x10⁻⁹, 0.5x10⁻⁹ and 0.6x10⁻⁹ [m² s⁻¹].

The order of magnitude difference between the quadratic and the exponential $D(\theta)$ functions is of concern. However if $D(\theta)$ is sharply quadratic around an approximately central minimum, then an exponential function would not be able to represent it. The large errors in two of the three replicates were consistent with the exponential function being inappropriate.

The minimum $D(\theta)$ results for the repacked sand obtained using the quadratic $D(\theta)$ functions fitted to the evaporation data (Figure 20), were similar to the minimum values of $D(\theta)$ shown in the study of Rose (1968). In the study of Rose (1968), the minimum value of $D$ was usually reached at a suction of about 1500 kPa. In Figure 20 the minimum value of $D$, for the quadratic $D(\theta)$ functions, was reached at suctions somewhat wetter than 1500 kPa, at approximately 700 to 800 kPa.
Soil water content $[m^3/m^3]$  

$D(\theta)$, calculated using the analysis of Passioura (1976) on the 500 kPa outflow experiment on repacked sand (Figure 20 and Figure 18), reached a constant minimum of $7 \times 10^{-9}$ [m$^2$ s$^{-1}$] and the exponential $D(\theta)$ function, from the optimization analysis from the 500 kPa outflow data, monotonically decreased to $5 \times 10^{-9}$ [m$^2$ s$^{-1}$]; these values are close to the constant minimum values of the three exponential $D(\theta)$ functions from the evaporation experiments. However the quadratic $D(\theta)$ function, from the optimization analysis of the 500 kPa outflow data, reached a lower minimum value of $2 \times 10^{-9}$ [m$^2$ s$^{-1}$].
In conclusion, the outflow experiments, when analysed using the analysis of Passioura (1976), mostly agreed with the $D(\theta)$ functions from the evaporation method, thereby validating the evaporation method. The outflow experiments, when analysed using the analysis of Passioura (1976), resulted in predominantly constant values of $D$ over the suction range where the soil is thought to be limiting the uptake of water by plant roots. This result provides justification for the use of a constant $D$ for modelling the flow of water to the plant roots. A constant $D$ of 10 or $20 \times 10^{-9}$ [m$^2$ s$^{-1}$] is appropriate for the undisturbed clay loam and the clay loam repacked to 1.3 g cm$^{-3}$. For the repacked sand, due to the variability in the minimum $D$ for the various analyses, a constant $D$ ranging from 1 to $7 \times 10^{-9}$ [m$^2$ s$^{-1}$] is appropriate. The point measurement of $D$, calculated from $k$ and $\delta\sigma/\delta\theta$, showed that the evaporation method may be unreliable at the wet end.
4.1 Introduction

For analysis of plant water uptake an experimental system is required that simultaneously measures both the plant transpiration rate, \( E \), and the pressure drop across the plant and soil in real time. The purpose of this chapter is to describe such an experimental system.

This system contains a plant where the roots and soil are located inside a pneumatic pressure chamber and the leaves are contained inside a transparent glass cuvette (see schematic Figure 21 and photo Figure 22). The water pressure in the leaf xylem can be raised to atmospheric pressure, reference zero, by applying a positive pneumatic pressure to the root chamber and observing when a freshly cut leaf is on the verge of bleeding.

The pneumatic pressure applied to the root chamber to bring the leaf xylem to atmospheric pressure is the pressure drop across the plant and soil that is equal to the negative water pressure in the leaf xylem if the positive pneumatic pressure were not applied. This pressure is termed balancing pressure, \( B \).

The system described below measures \( B \) and \( E \) by calculating the product of the air flow rate into the glass cuvette and the change in humidity of the air as it passes through the cuvette. The preparation of the plants and pots used in the system is described in Chapter 5 section 5.2.1.
Figure 21 Schematic of equipment used to measure pressure drop across the plant and soil. The infrared emitter and detector form a position sensor that monitors the position of the meniscus inside the capillary tube. The position sensor is connected to a pressure controller that regulates the pressure inside the pressure chamber. Drawing not to scale.
Figure 22 Photo of plant fully assembled with roots in pressure chamber, (a), and leaves inside cuvette, (b), showing light source and screens, (c) to regulate light intensity.

4.2 Pressure control of root chamber

The pressure inside the root chamber is controlled by a feedback mechanism that responds to the increase or decrease in pressure of the leaf xylem. This is detected by maintaining hydraulic continuity between the xylem vessels in the cut leaf and a capillary tube (radius of 330 µm) positioned inside a position sensor with an infra-red emitter and detector. When hydraulic continuity exists between the leaf xylem and
capillary tube, the meniscus inside the capillary tube rises and falls when the pressure in the xylem sap is positive or negative with respect to the pressure of the atmosphere.

The position of the meniscus inside the capillary tube is detected by an infrared emitter and detector (Figure 21) inside a position sensor. The position sensor is connected to a pressure controller that adjusts the pressure inside the root chamber. If the meniscus is high the pressure inside the root chamber is decreased; conversely if the meniscus is low the pressure inside the root chamber is increased.

Hydraulic continuity must be maintained between the capillary tube, positioned inside the position sensor, and the leaf xylem for the system to work. This is achieved using a custom built sensor positioned on the end of a cut leaf, referred to as the leaf sensor. A schematic of the leaf sensor is shown in Figure 23 and a photo is shown in Figure 24.

Figure 23 and Figure 24 show the leaf positioned inside a bulldog clip. Inside the bulldog clip the cut leaf sits inside a pouch made of filter paper (Whatman 42) which is sealed with gaffer tape to prevent evaporation. The filter paper is folded in half providing a crease for the leaf to sit. A small slit is cut in the filter paper and a nylon tube (shown protruding from the left of the bulldog clip in Figure 24) slides underneath the edge of the filter paper and through the small slit. The inside diameter of the nylon tube is 0.25 mm and the outside diameter is 0.75 mm. The nylon tube is located inside the filter paper pouch. Gaffer tape is wrapped around the filter paper pouch and nylon tube; securing the nylon tube underneath the filter paper and minimizing evaporation from the filter paper.
Figure 23 Schematic cross section (left) and front view (right) of custom built sensor. See text and Figure 24 for description. Drawing not to scale.
Minimizing evaporation from the filter paper is important to prevent loosing hydraulic continuity between the leaf xylem and capillary tube, and thereby maintain the leaf sensor as a null instrument. The small nylon tube is connected to the capillary tube located between the infrared emitter and detector shown in Figure 21. To prevent leaf damage the bulldog clip is prestressed and foam is positioned between the gaffer tape and bulldog clip.
The filter paper is maintained close to saturation, but at a slight suction, as the leaf xylem is maintained close to atmospheric pressure. The capillary tube inside the infrared emitter and detector exerts a slight suction (approximately -5 cm) on the water held in the filter paper. So that when the pressure inside the leaf xylem is increased slightly above that of atmospheric, the suction of the capillary tube inside the position sensor causes the meniscus to rise; the pressure control system then decreases the pressure in the root chamber accordingly. Conversely when the pressure inside the leaf xylem falls just below that of atmospheric, the water content of the filter paper decreases slightly and the meniscus inside the position sensor falls and the pressure inside the root chamber is then increased.

4.3 Adjusting pressure in root chamber according to leaf xylem water potential

Figure 21 shows schematically the feedback system used to regulate the pressure inside the root chamber in order to maintain the water pressure in the leaf xylem close to atmospheric pressure. This section describes the function of the pressure controller shown in Figure 21.

The display panel of the pressure controller used to regulate the pressure inside the root chamber is shown in Figure 25. The pressure inside the chamber is measured by a pressure transducer and displayed in kPa (G in Figure 25). The signal from the infrared emitter and detector (shown in Figure 21), indicating the position of the meniscus inside the glass capillary tube, is displayed on the sensor meter (B in Figure 25); either above (+) or below (-) the central position (zero). The sensitivity knob (C in Figure 25) sets the factor by which the deflection of the sensor meter is multiplied to give a change in pressure to be added or subtracted to the set pressure and accumulation on the integrator. The integrator (D in Figure 25) raises (or lowers) the set point (pressure) according to whether the sensor meter reads low or high. The rate of accumulation (E in Figure 25) is set by the knob immediately under the integrator meter. The pressure set knob (A in Figure 25) sets the pressure when both the sensor and integrator meters read zero or are switched off and sets the base pressure which is adjusted according to the sensor meter, sensitivity knob, integrator meter and integrator knob. The rate at which gas is supplied or let escape from the chamber is set by F in Figure 25 and the needle valve shown in Figure 26.
The pressure inside the root chamber, $G$, is given by $A + B \times C + D$. The function of the six components, $A$, $B$, $C$, $D$, $E$ and $F$ in Figure 25, used to regulate the balancing pressure inside the root chamber in response to the water pressure in the leaf xylem is shown in the process diagram in Figure 26.

![Figure 25 Pressure controller used to regulate pneumatic pressure inside root chamber. “A” is the pressure set, “B” is the sensor meter displaying the position of the meniscus inside the capillary tube, “C” is the sensitivity knob that sets the factor to be multiplied by the sensor reading and added (or subtracted) to the pressure set, “D” is the integrator that accumulates (or dissipates) pressure depending if the sensor meter is high or low, the rate that the integrator accumulates or dissipates pressure is set by the integrator knob “E”. The pressure inside the root chamber is given by $A + B \times C + D$ and displayed in kPa, indicated by “G”. “F” is the bypass delay that, along with the needle in Figure 26, sets the rate at which gas is supplied to or released from the pressure chamber.](image)

The comparator in Figure 26 compares the output from the pressure transducer ($\epsilon$), that measures the pressure in the pressure chamber, with a variable set point ($\delta$), and then sends a logical signal (reduce pressure if $\epsilon > \delta$, otherwise increase it) to the 3-way solenoid valve. The set point ($\delta$) is the sum of three components: the basal value or pressure set ($\alpha$ in Figure 26, $A$ in Figure 25) which is the balancing pressure set by the operator; a jump value ($\beta$) which changes sign according to the meniscus sensor output ($B$ in Figure 26 and Figure 25) and is subject to the sensitivity setting ($C$ in...
Figure 26 and Figure 25); and the output of an integrator (γ) that is set by an adjustable ramp rate (E in Figure 26 and Figure 25).

The rate at which gas is supplied or released from the pressure chamber via the 3-way solenoid valve is set by the needle valve or bypass solenoid. Further opening the needle valve increases the flow rate to the pressure chamber and pressure transducer which increases the rate of 3-way solenoid ON/OFF cycling; closing the needle valve decreases this rate. In practice the needle valve is set to enable the solenoid valve to cycle once every five to ten seconds depending on how fast the balancing pressure is changing.

Opening the bypass solenoid overrides the needle valve, increasing the flow rate to the root chamber, and causes rapid 3-way solenoid ON/OFF cycling. When the bypass solenoid is set on automatic, the rate at which it is opened is set by the bypass delay knob (F in Figure 25). Each time the 3-way solenoid valve changes state a timer is reset and after a period of elapsed time, that is set by the bypass delay knob (F in Figure 25), the bypass solenoid is opened. When the meniscus height is falling or rising rapidly, the bypass delay is set low, thereby enabling the control system to rapidly adjust the pressure in the root chamber. Conversely, when the meniscus height is only changing slowly and the pressure in the root chamber requires only slow adjustment, the bypass delay is set high and the consumption of gas is reduced.

Figure 26 also shows that the high pressure gas supply used in the pressure chamber is a mixture of compressed air and nitrogen; this is to maintain the partial pressure of oxygen at about 21 kPa regardless of the total operating pressure. The operation of the mixing solenoid used to regulate the compressed air and nitrogen mixture is explained in section 4.4.
Figure 26 Schematic of system used to control pneumatic pressure in root chamber. The 3-way solenoid valve is used either to supply high pressure gas to the pressure chamber or let gas escape from the chamber, at a rate controlled by the needle valve or bypass solenoid. The state of the 3-way solenoid valve is set by a comparator that compares the output of a pressure transducer (connected to the pressure chamber) to that of a variable set point. The mixing solenoid is used to maintain the partial pressure of oxygen at about 21 kPa by opening the air line for four seconds only, then opening the nitrogen line for four seconds multiplied by the ratio of the current balancing pressure to atmospheric pressure, i.e. 100 kPa (see section 4.4).
4.4 Regulating air and nitrogen supply to root chamber

To maintain the partial pressure of oxygen inside the root chamber at about 21 kPa, (approximately that of the atmosphere) regardless of the total operating pressure, the high pressure gas supply is a mixture of compressed air and nitrogen. The partial pressure of oxygen is 21 kPa in air at 100 kPa (approximate atmospheric pressure at sea level), however when air is compressed to, say, 500 kPa the partial pressure of both oxygen and nitrogen is increased a factor five; to 105 and 390 kPa respectively. Hyperoxidation in the root chamber, which could damage the plant, is avoided by mixing nitrogen gas with the compressed air. The other major components of air, argon (<1%) and carbon dioxide (<0.04%), are ignored as their concentrations are considered negligible.

The mixing solenoid opens the air line for four seconds only, and then the nitrogen line is opened for four seconds multiplied by the ratio of the current balancing pressure (pressure transducer output $\varepsilon$ in Figure 26) to atmospheric pressure (i.e. 100 kPa). The mixing solenoid shown in Figure 26 mixes according to time not volume, and maintaining the partial pressure of oxygen close to 21 kPa relies on the air and nitrogen gas supplies being at the same pressure.

4.5 Atmospheric conditions inside cuvette

The plant leaves inside the cuvette are illuminated horizontally by a 400 W metal halide lamp that provides a photosynthetic photon flux density ranging between 70 and 500 $\mu$mol m$^{-2}$ s$^{-1}$. The photon flux density is adjusted by removing and replacing wire mesh screens (0.5 mm wire thickness, 1 mm$^2$ aperture) from in front of the lamp. With both screens in front of the light source the photon flux density is approximately 75 $\mu$mol m$^{-2}$ s$^{-1}$ and 240 and 500 $\mu$mol m$^{-2}$ s$^{-1}$ when the first and second screens are removed, respectively.

The system used to measure plant transpiration rate and also adjust the humidity and air speed flowing through the cuvette is shown schematically in Figure 28. The air flow rate passing through the cuvette ranges from 1.8 to 12 L min$^{-1}$ and is measured at the air inlet to the cuvette with a Hastings mass flow meter (model EALL-5KP). The humidity of the
The humidity of air flowing into the cuvette is varied by adjusting the addition of water into the air flowing to the cuvette. The ingoing air passes through sintered glass (Figure 27) and its water content is increased according to the amount of water on the sintered glass. The minimum humidity level is obtained by bypassing the sintered glass so that ambient air is passed through the cuvette, which has been compressed to 300 kPa and is therefore quite dry.

![Diagram](image)

**Figure 27 System, referred to as “Humidifier” in Figure 28, for conditioning humidity of air that passes through the cuvette. Flow rate of peristaltic pump determines the amount of water added to the air-stream. The temperature depression due to evaporation from the sintered glass is buffered by the water jacket. Drawing not to scale.**
Figure 28 Schematic of system used to measure plant transpiration rate and also adjust humidity and speed of air flowing through cuvette.
The atmospheric conditions inside the cuvette are varied to increase the evaporative demand in a series of steps (Table 3). At each step the air flow, humidity and light intensity settings are maintained until the control system is steady, as evidenced by the stable plot of the meniscus with time, and a reliable measurement of the transpiration rate has occurred; this usually takes 30 min. Once the greatest evaporative demand setting is obtained or the balancing pressure is approaching the pneumatic pressure limit of the equipment, 2000 kPa, the evaporative demand is decreased and the sequence of steps is reversed to investigate any hysteresis.

Table 3 Transpiration demand settings imposed on the plant inside the cuvette. Each setting is usually applied for 30 min to enable steady state conditions. When the greatest evaporative demand is obtained the sequence is reversed to investigate any hysteresis.

<table>
<thead>
<tr>
<th>Setting No.</th>
<th>Air flow [L min⁻¹]</th>
<th>Humidity [g m⁻³]</th>
<th>Light intensity [µmol m⁻² s⁻¹]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.8 to 2.0</td>
<td>12.0 to 15.0</td>
<td>75</td>
</tr>
<tr>
<td>2</td>
<td>2.4 to 2.6</td>
<td>6.5 to 7.0</td>
<td>75</td>
</tr>
<tr>
<td>3</td>
<td>3.0 to 3.0</td>
<td>6.5 to 7.0</td>
<td>240</td>
</tr>
<tr>
<td>4</td>
<td>3.5 to 3.6</td>
<td>4.5 to 5.5</td>
<td>240</td>
</tr>
<tr>
<td>5</td>
<td>4.0 to 4.5</td>
<td>4.5 to 5.5</td>
<td>500</td>
</tr>
</tbody>
</table>

4.6 Temperature control and measurement

The temperature of the pressure chamber that contains the soil core and plant roots is controlled at a constant temperature of 11°C; this is a similar temperature to that experienced by the roots growing in the field. The schematic in Figure 29 shows that the electronic temperature control unit is set to the desired pressure chamber temperature and pumps 1 and 2 circulate water, that is maintained at the desired pressure chamber temperature, inside the pressure chamber. The error signal from the platinum thermometer 100 (PT100) inside the water bath is used by the temperature control unit to activate pump 3, which pumps ethanol through the cooling coil located inside the water bath. The heat pump maintains the ethanol at 5°C.
Figure 29 System used for temperature control of pressure chamber. Temperature control unit is set to desired pressure chamber temperature and pumps 1 and 2 circulate water, maintained at desired set temperature, inside pressure chamber. Error signal from PT100 inside water bath is used by heat pump to activate pump 3 which pumps ethanol through cooling coil inside water bath. Ethanol is maintained at 5°C by temperature controller unit. Drawing not to scale.

Copper constantan thermocouples are positioned in the following locations; two inside the pressure chamber, one adjacent the system measuring the ambient (laboratory) temperature, one at the base of the plant close to the pressure seal inside the cuvette and another is positioned on (touching) the youngest, fully expanded leaf inside the cuvette.

The thermocouple touching the leaf is used to estimate stomatal conductance and is held in place with masking tape as shown in Figure 30. The masking tape is nominally 5 mm in width; the length is dependant on the width of the leaf. The thermocouple wire is positioned parallel with the leaf to enable attachment to the leaf, with the junction touching the leaf. The junction of the thermocouple must be touching the leaf for a reliable leaf temperature measurement, as the boundary layer in temperature between the leaf and the atmosphere adjacent to the leaf is small. The masking tape is positioned
5 mm back from the junction, perpendicular to the leaf and thermocouple wire. The masking tape is folded over so that it sticks to itself and effectively clamps the thermocouple and leaf. This enables removal of the thermocouple after the experiment without damaging the leaf.

Figure 30 Schematic of copper constantan thermocouple positioned on (touching) youngest, fully expanded leaf. Note the use of masking tape as a clamp to hold the thermocouple in position. Left hand diagram is a cross sectional view from above. Drawing not to scale.

The thermocouples are referenced against a 4-wire PT100 temperature sensor by positioning the reference junctions of the thermocouples inside an isothermal block. The temperature of the isothermal block is measured with a 4-wire PT100 temperature sensor and added to the temperature measurement of the thermocouples.

A 4-wire PT100 is a resistance thermometer that exploits the known change in electrical resistance of platinum (Pt) with temperature. PT100 sensors have a nominal resistance of 100 ohms at 0°C. The name “4-wire” denotes that four wires are used to measure the resistance of the sensor. This increases the measurement accuracy as the resistance in the lead wires is accounted for by supplying a very low current excitation (< 1 mA to avoid heating the measuring element) and measuring the voltage differential. The resistance of the measuring element is then calculated using Ohm’s Law (45).
\[ I = \frac{V}{\Omega} \]  \hspace{1cm} (45)

Where \( I \) is current [Amperes], \( V \) is voltage [volts] and \( \Omega \) is resistance [Ohms].

The temperature of the 4-wire PT100 is calculated from the resistance output by fitting a 2\textsuperscript{nd} order polynomial (46) to standard PT100 temperature versus resistance data obtained from Omega (www.omega.com).

\[
PT100 = -246.063 + 2.36249\times\Omega + 9.81597\times10^{-4}\times\Omega^2
\]  \hspace{1cm} (46)

The temperature of the thermocouples is calculated by fitting a 2\textsuperscript{nd} order polynomial to standard temperature versus millivolt data for copper constantan thermocouples (www.omega.com). The temperature of the PT100, located in an isothermal block with the reference junction of the thermocouples, is then added to the temperature of the thermocouples (47).

\[
TC = -0.014757 + 25.860411\times mV - 0.640368\times mV^2 + PT100
\]  \hspace{1cm} (47)

Where \( TC \) is the temperature [°C] of the measurement thermocouple junction and \( mV \) is the millivolt output representing the voltage differential between the reference and measurement thermocouple junctions.

### 4.7 Logging outputs and measurement of transpiration

The output signal from the instruments used in the system to measure plant water uptake and balancing pressure, listed in Table 4, are logged every second using a Campbell Scientific CR3000 data logger. The data logger is programmed to return the data in real time.

**Table 4** List of instruments used in system to measure plant water uptake and balancing pressure. Where “sample” and “reference” denote outgoing and ingoing air-streams respectively.

<table>
<thead>
<tr>
<th>Instrument</th>
<th>4-wire PT100 for determining thermocouple temperatures</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5 copper constantan thermocouples</td>
</tr>
<tr>
<td></td>
<td>“sample” and “reference” dew-point hygrometers</td>
</tr>
<tr>
<td></td>
<td>2-wire PT100 from temperature controller</td>
</tr>
<tr>
<td></td>
<td>Flow meter [0-5 L min(^{-1})]</td>
</tr>
<tr>
<td></td>
<td>Flow meter [0-50 L min(^{-1})]</td>
</tr>
<tr>
<td></td>
<td>Pressure transducer (balancing pressure)</td>
</tr>
<tr>
<td></td>
<td>Infrared emitter and detector (meniscus)</td>
</tr>
</tbody>
</table>
The output from each dew-point hygrometer was calibrated with the temperature output from the 4-wire PT100; a schematic of the system is shown in Figure 31. The 4-wire PT100 was positioned inside a vessel on a sintered glass surface containing water and an air-stream was bubbled through the sintered glass, thereby conditioning the air-stream to the dew-point temperature of the water. The air-stream was then passed through the dew-point hygrometer and both instruments were logged. The vessel containing the sintered glass surface was positioned in a temperature controlled water bath to enable calibration across a range of temperatures.

![Figure 31 Schematic of system used to calibrate the dew-point hygrometers with PT100 temperature measurement. Drawing not to scale.](image)

The calibration results are shown in Figure 32. The dewpoint hygrometers are both General Eastern model Dew-10-xx1 (accuracy +/−0.5°C, repeatability +/-0.05°C); however the “reference” dewpoint hygrometer is connected to a visual display unit and the output is in °C.
The accuracy of the two dew-point hygrometers was tested by comparing the cumulative evaporation from a petri dish filled with water measured in two different ways; (1) calculated from the difference in dewpoint between the ingoing and outgoing air-streams and the flow rate of air through the cuvette and (2) the change in weight of the petri dish. The dewpoint hygrometers underestimated the evaporation by 2%, giving confidence that the dewpoint hygrometers were sufficiently accurate.

The transpiration rate is calculated every second as the product of the air flow rate through the cuvette and the difference in water content between the ingoing and outgoing air-streams. The vapour pressure of each respective air-stream is calculated from the dewpoint temperature using the Goff-Gratch formula (48) (Goff & Gratch 1946).

\[
\log_{10} p_0 = -7.90298(T_s / T - 1) + 5.02808 \log_{10}(T_s / T) \\
-1.3816 \times 10^{-7} \left[ 10^{11.344(T / T - 1)} - 1 \right] \\
+8.1328 \times 10^{-9} \left[ 10^{8.19149(T / T - 1)} - 1 \right] + \log_{10} P_e
\]

Where \(p_0\) is the saturation vapor pressure over liquid water [hPa], \(T\) is the absolute (thermodynamic) temperature [°K], \(T_s\) is the steam point temperature [373.16°K] and \(P_e\) is the saturation pressure of liquid water at steam point temperature.
The plant transpiration rate, $E \text{[g s}^{-1}]$, is calculated using (49) where $p_{\text{diff}}$ is the difference in vapour pressure between the ingoing and outgoing air-streams, $p_{\text{atm}}$ is the atmospheric pressure (assumed to be 1013 hPa), the ratio of $p_{\text{diff}}$ to $p_{\text{atm}}$ is the mole fraction of water vapour, $V_m \text{[m}^3 \text{mol}^{-1}]$ is the molar volume of an ideal gas at the given temperature, $M_m \text{[g mol}^{-1}]$ is the molar mass of water and $F_{\text{air}} \text{[m}^3 \text{s}^{-1}]$ is the flow rate of air through the cuvette.

$$E = \frac{p_{\text{diff}} \cdot F_{\text{air}}}{p_{\text{atm}} \cdot V_m} \cdot M_m$$

(49)

The stomatal conductance, $g \text{[mol m}^{-2} \text{s}^{-1}]$, is estimated using (50) where $LA \text{[m}^2]$ is the leaf area and $VPD \text{[hPa]}$ is the estimated vapour pressure deficit between the stomatal cavity inside the leaf and ambient air inside the cuvette. The vapour pressure inside the stomatal cavity of the leaf is estimated using the measurement of leaf temperature and (48), where saturated conditions inside the stomatal cavity are assumed.

$$g = \frac{p_{\text{atm}} \cdot F_{\text{air}}}{V_m \cdot VPD} \cdot \frac{1}{LA}$$

(50)

### 4.8 Summary

This chapter has described a system that simultaneously measures the pressure drop across the plant and soil and the plant transpiration rate; non-destructively and in real time. Thereby enabling testing of specific hypotheses as to what may be limiting the extraction of water from the soil by the plant roots. The roots are located inside a pressure chamber and the leaves are contained inside a transparent glass cuvette. The water pressure in the leaf xylem is raised to atmospheric pressure, reference zero, by applying a positive pneumatic pressure to the root chamber and observing when a freshly cut leaf is on the verge of bleeding. This pressure is called balancing pressure.

The balancing pressure is the pressure drop across the plant and soil, that is equal to the negative water pressure in the leaf xylem, if the positive pneumatic pressure were not applied. The transpiration rate of the plant can be varied across a wide range and is measured by calculating the product of the air flow rate into the glass cuvette and the change in humidity of the air as it passes through the cuvette.
5.1 Introduction

There are various hypotheses as to why the extraction of seemingly available water from the subsoil is incomplete (see Chapter 1). These include: that the flow rate of water through the soil to individual well-distributed roots is limited by the hydraulic properties of the soil; that there is a large interfacial resistance to the flow of water between the soil and root, possibly arising from root shrinkage and vapour gaps; that the roots are clumped, so that water must move long distances to them; and that there may be an osmotic choke if solutes excluded by the root as the water enters increase in concentration at the root surface.

The hypotheses implicating soil resistances are testable using measured soil hydraulic properties over the suction range of interest, real time experimental measurements of the plant transpiration rate and the xylem water potential and the total root length. The experimental measurements can then be compared to numerical calculations of eq. 1 to test if the soil can become limiting.

The flow of water out of a plant leaf is controlled by the aerial environment around the leaf, until the stomata start to close. Therefore the greatest resistance to flow in the hydraulic path from the soil through the roots, plant, leaves and into the atmosphere is at the stomata. Figure 33, shows modelled data, using numerical calculations of eq. 1, of two hypothetical experiments; (1) the evaporative demand is progressively increased (rising $E$ phase), at a rate of $1 \ [\mu g \ s^{-1}]$ per minute from 25 to 600 $[\mu g \ s^{-1}]$ (shown as the solid lines in Figure 33(a) and (b)), and then decreased (falling $E$ phase), at a rate of $1 \ [\mu g \ s^{-1}]$ per minute from 600 to 25 $[\mu g \ s^{-1}]$ (shown as the dashed lines in Figure 33(a) and (b)) and (2) the evaporative demand is progressively increased, at a rate of $1 \ [\mu g \ s^{-1}]$ per minute from 25 to 1245 $[\mu g \ s^{-1}]$, shown in Figure 33(c). The parameters were the same for both modelling runs, where a root length density (length of root per volume of soil), $L_v$, of 0.65 $[cm \ cm^{-3}]$ and a constant $D$ of $1 \times 10^{-9} \ [m^2 \ s^{-1}]$ were used.
Figure 33(a) shows the balancing pressure, $B$, which is the null measurement of the xylem water potential (see Chapter 4) and represents the pressure drop across the plant and soil (relative to atmospheric pressure), as a function of the transpiration rate, $E$, which is the flow rate of water through the plant, and the model output of $\tau$ at the root surface.

In wet soil, where $D(\theta)$ is large, at low to moderate $E$ the linear slope of successive points of $B(E)$ represents the hydraulic resistance of the plant, $R_{\text{Plant}}$ [kPa s $\mu g^{-1}$]; this is shown in Figure 33(a) when $E$ is increasing from 25 to 200 [$\mu g$ s$^{-1}$]. The product of $R_{\text{Plant}}$, which in this model is assumed constant, and $E$, is the pressure drop across the plant, $\Delta P_{\text{Plant}}$.

When the soil is at equilibrium, the pressure drop across the air water interface of all the water filled pores in the bulk soil is $\tau_{\text{Bulk}}$; which is shown as the intercepts on the $B$ axis in Figure 33(a) as $\tau_{\text{Bulk}}$ at the start and finish of the experiment.

However, the intercepts of the rising and falling $E$ phases, only provide estimates of $\tau_{\text{Bulk}}$ on the proviso that the difference in $\tau$ at the root surface and in the bulk soil, $\Delta \tau$, is negligibly small (at low $E$). This proviso extends further to the estimate of $R_{\text{Plant}}$, which is only valid if $\Delta \tau$ is negligibly small. Then, when the soil is wet, across a medium to low range of $E$, when the $B$ is a linear function of $E$, using eq. 51, the data can be interpreted with a simple linear function:

$$B = R_{\text{Plant}} \times E + \tau_{\text{Bulk}} \quad (51)$$

Where $R_{\text{Plant}}$ and $\tau_{\text{Bulk}}$ are constants representing the hydraulic resistance of the plant [kPa s $\mu g^{-1}$] and the soil water suction in the bulk soil [kPa] respectively.

Figure 33(b) shows all of the pressure drops that can be summed together and equated with $B$:

$$B = \Delta P_{\text{Plant}} + \tau_{\text{Bulk}} + \Delta \tau \quad (52)$$

From close inspection of Figure 33(a) and (b), it is clear that the slope of $B(E)$, at low to moderate $E$, may appear linear when $\Delta \tau$ is no longer negligibly small; then the intercept
of increasing $E$, and therefore $\tau_{\text{Bulk}}$, will be underestimated, and $R_{\text{Plant}}$ will be overestimated.

Figure 33(b) shows that $\tau_{\text{Bulk}}$, at low $E$ during the rising $E$ phase, is close to the intercept of $B(E)$ for the rising $E$ phase and $\tau$ at the root surface (where the latter two are shown in Figure 33(a)), and $\Delta \tau$ is negligibly small. Also, $\tau_{\text{Bulk}}$ (Figure 33(b)) at low $E$ during the falling $E$ phase, is also close to the intercept for the falling $E$ phase and $\tau$ at the root surface for the falling $E$ phase (Figure 33(a)); however the intercept for the falling $E$ phase in Figure 33(a) will slightly overestimate $\tau_{\text{Bulk}}$, because Figure 33(b) shows that $\Delta \tau$ is not negligibly small at low $E$ for the falling $E$ phase.

In Figure 33(c), the hypothesis that the combination of low $D(\theta)$ and low $L_v$ at low $\theta$ is the main resistance to the uptake of water by plant roots, is demonstrated by the singularity of $B$ and $\Delta \tau$ at large $E$. 
This chapter examines the hypothesis that the soil is the main resistance to the extraction of water by the plant roots, owing to a combination of low $L_v$ and low $D(\theta)$ at low $\theta$. This hypothesis was investigated by analysing plant water uptake at the single plant scale on three soil treatments; undisturbed clay-loam, repacked clay-loam and repacked sand. These soil types are representative of two major wheat growing areas in Australia; the sand is from the wheat belt in Western Australia and the clay-loam is from the NSW
southwest slopes in south-eastern Australia. The aim of this chapter is to further our understanding of plant water uptake by comparing measured experimental results with a model that solves the diffusion equation for radial flow of water to a plant root.

5.2 Methods

5.2.1 Plant water uptake experiment

The measurements of plant water uptake or transpiration rate, \( E \), and balancing pressure, \( B \), were made using the experimental method described in Chapter 4.

Three of the soil types used for measurements of \( D(\theta) \) and soil water retention described in Chapter 3 were used: undisturbed clay-loam, repacked clay-loam and repacked sand. The undisturbed and repacked clay-loam was obtained from the same location described in section 3.2.1. The repacked clay-loam was sieved to less than 2 mm and repacked to a bulk density of 1.3 g cm\(^{-3}\). The sand was collected from the Western Australian wheat belt, sieved to less than 2 mm and repacked to a bulk density of 1.6 g cm\(^{-3}\). The hydraulic properties of these soils are presented in section 3.3.

The repacked pots were made of PVC and were 20 cm in length by 8.6 cm inside diameter (ID). The soil was packed in 1 cm increments using an arbor press with a cylindrical piston just smaller than the ID of the pot, thereby ensuring that the bulk density was uniform with depth. Three replicates of the repacked clay-loam and two replicates of the repacked sand were used for plant water uptake experiments.

The undisturbed soil was sampled by firstly exposing an undisturbed horizontal shelf of soil at 20 cm below the soil surface and then using a hydraulic press to force a stainless steel cylinder, 20 cm in length by 8.6 cm ID, vertically into the soil (Figure 34). The stainless steel cylinder, containing the undisturbed soil sample, was then extracted by removing the soil from around the cylinder. Plant water uptake experiments were undertaken on two replicate undisturbed soil cores.
The two replicate undisturbed cores were supplied with 4 mL each of Hoagland’s (Hoagland & Arnon 1950) A and B stock solution augmented with phosphorus in the form of 3 mL of 0.5 Molar K$_2$H$_2$PO$_4$ (75 mg P). Each of the three solutions were applied to a different location on the top of the core at three points equal distance apart on the pots circumference. The concentrated solutions were applied because many Australian subsoils are low in nutrients. The solutions were applied as point sources on the top edges of the core, thereby allowing the roots to access the nutrients without the nutrient concentration becoming toxic to the plant, and without disturbing the low nutrient status of the bulk of the soil.

All of the repacked pots were leached with 1000 cm$^3$ (about twice the pore volume) of half strength Hoagland’s solution (Hoagland & Arnon 1950) (26 kPa osmotic pressure).
The repacked clay-loam and the repacked sand replicates were equilibrated at a suction of 10 kPa at the base of each pot. The first and second replicate cores of undisturbed clay-loam soil were equilibrated at suctions of 10 and 20 kPa at the base respectively. After each core was equilibrated at the desired suction, the base of each core was then covered with plastic petri dishes containing four 2 mm diameter holes to allow gas exchange.

A metal cap was fitted to the top of each core. A small hole located in the centre of the metal cap allowed the cores to be sown with a germinated Janz wheat seed (*Triticum aestivum*) (Figure 35). The seed was sown by positioning the narrow end of a cone shaped cut in half 55 mm diameter filter paper into the small hole in the metal cap. The filter paper cone was positioned in the hole using a small rod. Then a 4 cm length by 1 cm diameter PVC tube was positioned inside the filter paper cone and secured with elastic bands and screws; for clarity, the elastic bands and screws were omitted from Figure 35. The PVC tube was then filled to about 3 to 5 mm from the top with fine sand. The sand was then wet and allowed to drain. The germinated seed was then positioned in the sand, covered and a little more water applied.

The PVC tube with newly sown seed was then covered with a non-transparent beaker containing a wet filter paper and kept in this dark, humid environment for two days or until the shoot emerged and looked viable. The plant and pot were then moved to a growth chamber maintained at a day/night temperature of 11°C with a daily photoperiod of 10 h and photon flux density of 350 µm m⁻² s⁻¹ maintained at the leaf level.
Seven days after sowing the PVC tube was removed and the sand gently washed away from the roots. The filter paper cone was removed and the excess sand gently washed away from the recess where the roots were growing. By this time typically three seminal roots were established in the soil core. The shoot, now usually at the two leaf stage, was then held upright with a small PVC tube and a wire stand that screwed into the metal cap. The PVC tube was 1 cm long and cut along the length to enable a firm fit around the base of the shoot.

The hole in the cap around the roots was sealed with silicone rubber (Sylgard 184) and a metal retaining plug positioned on the seal around the roots (see Figure 36). This formed the gas tight seal.
Figure 36 Photo of a plant growing in a pot with a pressure seal to enable pressurization of the root system. The metal plug (a) and silicone rubber pressure seal (b) are at the base of the shoot. The metal cap (c), metal plug and silicone rubber pressure seal collectively provide a pressure seal between the soil and root system and the shoot. The root system and soil are located inside the metal pot (d).
The leaf area was measured using a ruler as the sum of the length x breadth x 0.78 for each leaf. Once the leaf area of the plants had reached about 20 cm$^2$, which usually took around 20 days after sowing, measurements of $B$ and $E$ were made every second or third day depending on how fast the plant was depleting the soil water. The leaf area was measured prior to each measurement of $B$ and $E$. The $B$ and $E$ were measured by subjecting the plant to the evaporative demand settings described in Table 3 and section 4.5.

Measurements of $B$ and $E$ were made on two replicates of undisturbed clay-loam soil. The most reliable measurements were made on the second replicate and these were analysed in detail.

The second replicate of undisturbed clay-loam soil (initially equilibrated at 20 kPa suction) was re-watered back to the original 20 kPa suction weight on two different days, thereby encouraging the roots to fully explore the soil in the pot. However no roots were evident at the base of the pot at any time. The plant was re-watered when, after each time, the plant had dried the soil to the extent that the xylem sap could no longer be raised to atmospheric pressure without exceeding the 2 MPa pressure limit of the equipment. The measurements of $B$ and $E$ were made on days 22, 24, 26, 28 and 30, at which point the plant was re-watered back to the 20 kPa suction weight. Measurements continued on days 32 and 36, the plant was then re-watered back to the 20 kPa suction weight and measurements made on days 38 and 40. In total, measurements of $B$ and $E$ were made on the second replicate of undisturbed clay-loam soil on ten different days and the plant was re-watered twice.

Three replicates of the repacked clay-loam soil were used for measurements of $B$ and $E$. The most reliable measurements were made on the third replicate and these were analysed in detail. The third replicate was measured on nine different days: every second day from 22 to 32 and then on days 32, 35 and 37. Following the experiment on day 37, the plant was re-watered back to the initial 10 kPa suction weight and the final $B$ and $E$ experiment made on day 39.
Measurements of \( B \) and \( E \) were made on two replicates of repacked sand. The most reliable measurements were made on the second replicate and these were analysed in detail. Measurements were made on six different days; 21, 24, 27, 29, 31 and 33. The experiment was attempted on day 35; however the soil was too dry and the pressure in the xylem sap could not be raised to atmospheric pressure without exceeding the 2.0 MPa pressure limit of the equipment. Following the attempted experiment on day 35 the sand was washed from the roots for determination of total root length and diameter.

For modelling the flow of water to the plant roots the length and diameter of the roots of the third and second replicates of the repacked clay-loam and the repacked sand, respectively, were determined using essentially the same method as Watt, Kirkegaard & Rebetzke (2005). Briefly, the roots were scanned on an Epson 1680 modified flatbed scanner (Régent Instruments Inc. Québec, CA) and the images analysed using WINRhizo software. The roots were preserved in 50% ethanol and prior to scanning, rinsed in water then stained in 0.05% toluidine blue (pH 4.4) for 3 min, rinsed again in water and then transferred to a clear perspex tray containing a shallow film of water for scanning.

Measuring the total root length is a destructive measurement and in order to preserve the undisturbed clay-loam soil cores for later study if needed, the total root length was not determined on the undisturbed clay-loam soil cores.

### 5.2.2 Plant water uptake model

As described in section 4.6 the root chamber was maintained at 11ºC during the plant water uptake experiments. Accordingly, for modelling the flow of water to the plant roots the soil hydraulic properties were scaled with temperature. The influence of temperature on soil water pressure head was scaled according to the influence of temperature on surface tension (Philip & de Vries 1957)

\[
\tau_T = \frac{\sigma_T}{\sigma_{\text{ref}}} \tau_{\text{ref}}
\]

(53)

where \( \tau_T \) and \( \tau_{\text{ref}} \) (\( \sigma_T \) and \( \sigma_{\text{ref}} \)) are soil suctions (surface tensions) at temperature \( T \) and reference temperature \( T_{\text{ref}} \) respectively. The soil water diffusivity was scaled according to the temperature effects on the dynamic viscosity and density of soil water (Constantz 1982)
\[ D_r(\theta) = \frac{\mu_{\text{ref}} \rho_r}{\mu_r \rho_{\text{ref}}} D_{\text{ref}}(\theta) \]

(54)

where \( D_r(\theta) \) and \( D_{\text{ref}}(\theta) \) are soil water diffusivities at the soil temperature, \( T \), and reference temperature, \( T_{\text{ref}} \), respectively; \( \mu_r \) and \( \mu_{\text{ref}} \) (\( \rho_r \) and \( \rho_{\text{ref}} \)) are the dynamic viscosity (density of soil water) at temperatures \( T \) and \( T_{\text{ref}} \) respectively.

The most reliable data sets of the undisturbed clay-loam, repacked clay-loam and the repacked sand were chosen to model; these were the second (day 40), third (days 35 and 37) and second (days 31 and 33) replicates respectively. In order to obtain agreement between the experiment and the model, the plant water uptake for both the repacked clay-loam and the repacked sand was modelled assuming that only 10\% of the measured total root length was extracting water; which is less than the 30\% of total root length estimated by Passioura (1980). Other studies have also postulated about the proportion of the total root length that is extracting water (Bristow, Campbell & Calissendorff 1984; Herkelrath, Miller & Gardner 1977b; Lang & Gardner 1970; Stirzaker & Passioura 1996).

The undisturbed clay-loam was modelled a number of times using a root length density, \( L_v \), that ranged from 4.0 to 0.5 [cm cm\(^{-3}\)], constant \( D \) values that ranged mostly from \( 10x10^{-9} \) to \( 1x10^{-9} \) [m\(^2\) s\(^{-1}\)] and assuming that the roots were confined to either the top 50, 75 or 100\% of the pot volume. Section 3.3 shows that \( D \) varies little with \( \theta \) in the range from 100 to 1500 kPa suction and that the minimum value of \( D \) measured on the undisturbed clay-loam soil ranged from about \( 10x10^{-9} \) to \( 1x10^{-9} \) [m\(^2\) s\(^{-1}\)]. However in an attempt to obtain agreement between the experiment and the model, when it was assumed that the roots were confined to the top 50 or 75\% of the pot and \( L_v \) was less than or equal to 1.0 [cm cm\(^{-3}\)], in some cases a constant \( D \) as great as \( 50x10^{-9} \) [m\(^2\) s\(^{-1}\)] was used.

For the repacked clay-loam and the repacked sand, the experimental data was modelled using both a variable \( D(\theta) \), measured for each respective soil in Chapter 3, and a constant \( D \), that gave the best fit of the model to the experimental data. The variable \( D(\theta) \) function used to model the re-packed clay-loam is shown in eq. 55 (also shown in Figure 15 as quadratic number two (section 3.3.3)) and constant values of \( D \), \( 2x10^{-9} \) and \( 8x10^{-9} \) [m\(^2\) s\(^{-1}\)] were used to model days 35 and 37 respectively. The variable \( D(\theta) \) function used to
model the plant water uptake for the repacked sand is shown in eq. 56 (also shown in Figure 16 quadratic number two (section 3.3.4)) and a constant $D$ of $1 \times 10^{-9}$ [m$^2$ s$^{-1}$] was used to model days 31 and 33. These functions are the average $D(\theta)$ of the replicate measurements shown in Figure 15 and Figure 16 (sections 3.3.3 and 3.3.4) in the range from 100 to 1500 kPa.

$$D(\theta) = 1.85 \times 10^{-9} - 2.89 \times 10^{-7} + 2.11 \times 10^{-6} \theta^2$$ (55)  
$$D(\theta) = 2.29 \times 10^{-9} - 8.65 \times 10^{-7} + 8.35 \times 10^{-6} \theta^2$$ (56)

The plant water uptake was modelled using the single root model, eq. 1 (section 1.3.1), subject to the initial and boundary conditions in eq. 57. The derivation of eq. 1 is shown in Appendix 3.

$$t = 0, \quad r_a \leq r \leq r_b, \quad \theta = \theta_i$$ 
$$t > 0, \quad r = r_a, \quad D(\theta) \frac{\partial \theta}{\partial r} = \frac{Q(r_a^2 - r^2)}{2r_a}$$ (57) 
$$t > 0, \quad r = r_b, \quad \frac{\partial \theta}{\partial r} = 0$$

Where $Q$ [m$^3$ m$^{-3}$ s$^{-1}$] is the rate of extraction of water from the soil, $r_a$ is the root radius and $r_b$ is the radial distance of the cylinder from the centre of the root that defines the watershed that the root is assumed to have exclusive access to: the average half distance between the roots. The root length density (length of root per volume of soil), $L_v$, was used to determine $r_b$ from eq. 2 (section 1.3.1).

The experimental data, including the transpiration rate, $E$ [m$^3$ s$^{-1}$], and the volume of the pot, $svol$ [m$^3$], was used to define the variable flux boundary condition at $r_a$:

$$Q = \frac{E}{svol}$$ (58)

The use of $Q$ implies that the water content inside the roots was also changing at the same rate as the soil. The estimate of $Q$ could be improved by subtracting the root volume from the pot volume, however the root volume was negligibly small relative to the pot volume.

Eq. 1 was solved numerically, using the finite difference method described in section 2.3, by setting:

$$R = \ln(r/r_a)$$ (59)
The log transform reduces the number of distance steps needed in the finite difference grid, especially if \((r_b/r_a)\) is large (Passioura & Cowan 1968). Substituting eq. 59 into eq. 1,

\[
\frac{\partial \theta}{\partial t} = \frac{e^{2R}}{r_a^2} \frac{\partial}{\partial R} \left( D \frac{\partial \theta}{\partial R} \right)
\]

(60)

with:

\[
\begin{align*}
t &= 0, & 0 \leq \ln\left(\frac{r_b}{r_a}\right) & \leq R_b, & \theta &= \theta_i, \\
t > 0, & R = 0, & D(\theta)\frac{\partial \theta}{\partial R} &= \frac{1}{2} Q\left(\frac{r_b^2}{2} - \frac{r_a^2}{2}\right) \\
t > 0, & R = R_b, & \frac{\partial \theta}{\partial R} &= 0
\end{align*}
\]

(61)

The model was tested by comparing, at each time step, two different calculations of the quantity of water, \(W [m^3]\), taken up by unit length of the root, at a given time, \(i\); (1) by subtracting the integral of \(\theta\) with radial distance from \(r = a\) to \(r = b\) at a particular time, \(t = i\), from the initial quantity of water in the soil, \(W_0\), (eq. 62), and (2) the integral of the flux with time at the root surface, \(r = a\), using eq. 63.

\[
W = W_0 - \left[ \int_{r=a}^{r=b} \pi \left( r_b^2 - r_a^2 \right) \theta(r,t) dr \right]_{t=i}
\]

(62)

\[
W = \left[ \int_{r=a}^{r=i} 2\pi r D(\theta) \frac{\partial \theta}{\partial R} dt \right]_{r=a}
\]

(63)

The accuracy of the program was tested by calculating the absolute difference between eq. 62 and eq. 63 and expressing it as a percentage of the quantity of water lost as calculated from the integral of water content with radial distance (eq. 62). This is called the flux error and should decrease as the size of the distance steps decreases (eq. 16), this is because the flux calculated using eq. 63 is proportional to \(\delta \theta/\delta r\) at \(r = a\), and the accuracy at which \(\delta \theta/\delta r\) is estimated increases with the number of distance steps. Also the accuracy of both \(\theta(r, t)\) and \(\delta \theta/\delta r\) at \(r = a\) will increase as the number of terms in the Taylor series expansion used to form the finite difference equation of eq. 1 increases, where the error is proportional to the order of the first term neglected (Crank 1975, p. 141). Figure 37 shows that the flux error for 1000 distance steps is <10% after 100 seconds and reduces to <1% after 1000 seconds; giving confidence that the algorithms used in the model are correct.
Figure 37 Flux error which is the absolute difference between eq. 62 and 63 expressed as a percentage of the quantity of water lost as calculated from the integral of water content with radial distance (eq. 62) for 10, 100 and 1000 distance steps for modelling flow of water to plant root. Time step, \( L_v \), \( D(\theta) \), \( E \) and \( \theta_i \) were the same for each run.

### 5.2.3 Soil hydraulic properties

The soil water diffusivity for each soil was measured using the method described in Chapter 2 and the soil water retention was measured using a standard tension table and pressure extraction apparatus. The soil water retention and soil water diffusivity, measured using the evaporation method (section 3.2.1), of the undisturbed clay-loam is presented in Figures 11, 12 and 13 (section 3.3.2), the repacked clay-loam is presented in Figure 15 (section 3.3.3) and the repacked sand in Figure 16 (section 3.3.4).

The van Genuchten (1980) function (eq. 64) was fitted to the soil water retention data for each soil using the minimum sum of squares criterion.

\[
\theta(\tau) = \theta_s + \frac{\theta_i - \theta_s}{[1 + |\tau|^{n_w}]}^m
\]  

(64)
Where $\theta_r$ and $\theta_s$ denote the residual and saturated soil water content respectively, $\alpha$ is the inverse of the air-entry value, $n$ and $m$ are shape coefficients. For the purpose of this study the five independent fitting parameters in eq. 64 are empirical coefficients and are shown for each soil type in Table 5.

The water retention data of the sand was described using two functions because it proved difficult to achieve an adequate fit across the entire range of suctions using only one function. The van Genuchten (1980) function (eq. 64) was fitted to the 10, 100 and 500 kPa points (fitted parameters shown in Table 5) and an exponential function fitted to the 500, 1000 and 1500 kPa points:

$$
\tau[\text{kPa}] = -39.58 + 39.29 \times 10^4 \times \exp(-124.19 \times \theta)
$$

For modelling the plant water uptake eq. 64 was used when $\theta \geq 0.0515$ [m$^3$ m$^{-3}$] and eq. 65 used when $\theta < 0.0515$ [m$^3$ m$^{-3}$].

Table 5 fitted parameters for van Genuchten (1980) retention function (eq. 64) for undisturbed clay-loam, repacked clay-loam and repacked sand. *Denotes that parameters are scaled to 13°C. The parameters for the repacked sand were only fitted to the 10, 100 and 500 kPa points.

<table>
<thead>
<tr>
<th></th>
<th>undisturbed clay-loam</th>
<th>repacked clay-loam*</th>
<th>repacked sand</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_r$</td>
<td>0.064</td>
<td>0.096</td>
<td>0.047</td>
</tr>
<tr>
<td>$\theta_s$</td>
<td>0.296</td>
<td>0.253</td>
<td>0.951</td>
</tr>
<tr>
<td>$\alpha$ [kPa$^{-1}$]</td>
<td>0.163</td>
<td>0.036</td>
<td>6.131</td>
</tr>
<tr>
<td>$n$</td>
<td>1.291</td>
<td>3.628</td>
<td>1.566</td>
</tr>
<tr>
<td>$m$</td>
<td>0.226</td>
<td>0.153</td>
<td>0.419</td>
</tr>
</tbody>
</table>

### 5.3 Results and discussion

#### 5.3.1 Undisturbed clay-loam

The most reliable measurements were made on the second replicate and these were analysed in detail. The measurements made on the first replicate are shown in Appendix 4. Plots of $B$ as a function of $E$ were created for each day that measurements were made. The slope and intercept, for each day, from the linear part of both increasing and decreasing $E$, and the leaf area are shown in Table 6.
Table 6 Second replicate of undisturbed clay-loam soil: slope, intercept and leaf area for each of the $B$ and $E$ experiments shown in Figure 38 and Figure 39. * denotes that plant was re-watered back to 20 kPa suction weight after days 30 and 36. Data from days 22 to 30 is plotted in Appendix 4. (up) and (down) denotes rising and falling $E$ phases respectively.

<table>
<thead>
<tr>
<th>Age (days from sowing)</th>
<th>Intercept (up) [kPa]</th>
<th>Slope (up) [kPa s $\mu g^{-1}$]</th>
<th>Intercept (down) [kPa]</th>
<th>Slope (down) [kPa s $\mu g^{-1}$]</th>
<th>Leaf area [cm$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>390</td>
<td>3.8</td>
<td>648</td>
<td>4.0</td>
<td>28.2</td>
</tr>
<tr>
<td>24</td>
<td>343</td>
<td>4.3</td>
<td>531</td>
<td>5.8</td>
<td>35.4</td>
</tr>
<tr>
<td>26</td>
<td>532</td>
<td>3.0</td>
<td>769</td>
<td>3.8</td>
<td>43.4</td>
</tr>
<tr>
<td>28</td>
<td>529</td>
<td>3.5</td>
<td>841</td>
<td>3.0</td>
<td>53.5</td>
</tr>
<tr>
<td>30</td>
<td>842</td>
<td>2.7</td>
<td>1335</td>
<td>2.0</td>
<td>65.2</td>
</tr>
<tr>
<td>*32</td>
<td>134</td>
<td>2.6</td>
<td>220</td>
<td>2.4</td>
<td>82.0</td>
</tr>
<tr>
<td>34</td>
<td>301</td>
<td>2.3</td>
<td>651</td>
<td>2.3</td>
<td>97.2</td>
</tr>
<tr>
<td>36</td>
<td>816</td>
<td>2.2</td>
<td>1074</td>
<td>2.1</td>
<td>115.9</td>
</tr>
<tr>
<td>*38</td>
<td>93</td>
<td>1.8</td>
<td>141</td>
<td>1.8</td>
<td>140.7</td>
</tr>
<tr>
<td>40</td>
<td>751</td>
<td>1.7</td>
<td>1113</td>
<td>1.4</td>
<td>162.5</td>
</tr>
<tr>
<td>*42</td>
<td>-51</td>
<td>1.8</td>
<td>92</td>
<td>1.5</td>
<td>189.6</td>
</tr>
</tbody>
</table>

The data from days 22 to 30 are plotted in Appendix 4. Figure 38 shows that on day 32 $B$ was a quasilinear function of $E$ for both the rising and falling $E$ phases that, because the intercepts of the rising and falling $E$ phases were the same and given the discussion of the hypothetical model in Figure 33, the slope of either line provides a reliable estimate of $R_{Plant}$.

Day 36 exhibits non-linear behaviour after the fourth point of the rising $E$ phase, thereby suggesting that the suction gradients close to the root may have been becoming appreciable. The data for the falling $E$ phase on day 36 was qualitatively similar to the hypothetical model shown in Figure 33, where $\Delta \tau$ decreases with falling $E$. That the final point of the falling $E$ phase, in Figure 38, is close to that of the initial point of increasing $E$, suggests that $\Delta \tau$ is relaxing and possibly that water is moving from lower in the core, where the roots had not yet explored, towards the top (analysed more fully later).
Transpiration rate, $E$ [µg s$^{-1}$]

Balancing pressure, $B$ [kPa]

Day 32, rising $E$ phase
- Day 32, falling $E$ phase
- Day 36, rising $E$ phase
- Day 36, falling $E$ phase

Figure 38 Second replicate of undisturbed clay-loam soil: $B$ as a function of $E$. The plant was re-watered back to 20 kPa weight after the experiment on days 30 and 36.

After the experiment on day 36 the plant was re-watered back to the original 20 kPa weight. Figure 39 shows that on day 38 the plots of rising and falling $E$ phases were approximately linear. That the plot of decreasing $E$ tracks back along the line of increasing $E$, suggest that $\Delta \tau$ was negligible.

On day 40, the first four measurements of $B$ with rising $E$ were linear, thereby suggesting that $\Delta \tau$ was negligible; thereafter $B$ accelerated with $E$ on a steeper slope, thereby suggesting that $\Delta \tau$ was no longer negligible. The plot of falling $E$, on day 40, is curvilinear: the curved shape may be due to either, or a combination of both; (1) relaxation of $\Delta \tau$, as the evaporative demand is reduced or (2) the movement of water from unexplored soil lower in the core to the plant roots located towards the top of the core. The second point is supported by the fact that the roots in the second replicate did not grow to the base of the pot and, for the first replicate, only one root was observed at the base of the pot.
Figure 39 Second replicate of undisturbed clay-loam soil: $B$ as a function of $E$. The plant was re-watered back to 20 kPa weight after the experiment on days 30 and 36.

The hypothesis that the soil is the main resistance to the extraction of water by the plant roots was tested by modelling the experimental data from day 40. For each modelling attempt the intercept for increasing $E$ (Table 6) was used to determine the initial soil water content for the pot, using the van Genuchten (1980) parameters (Table 5), and the slope of increasing $E$ (Table 6) was used to estimate $R_{\text{plant}}$.

Figure 40 shows the experimental data from day 40 (shown in Figure 39) modelled assuming that the roots occupied the total pot volume and using constant values of $D$ that range from $10 \times 10^{-9}$ to $4 \times 10^{-9}$ [m$^2$ s$^{-1}$] for $L_r = 1.0$ [cm cm$^{-3}$]. Each of the modelling attempts shown in Figure 40 underestimated $B$ when $E$ is decreased and when $D = 4 \times 10^{-9}$ [m$^2$ s$^{-1}$] (Figure 40(f)). The $B$ was overestimated by the model for all except the first two points when $E$ was increasing. Clearly, the modelling attempts shown in Figure 40 do not show the acceleration of $B$ with $E$, at large $E$. 

106
Figure 40 Second replicate of undisturbed clay-loam soil: model of experimental data from day 40 (from Figure 39). The data was modelled assuming the roots had explored the total pot volume and using a range of constant values of $D$ [m$^2$ s$^{-1}$] for $L_v = 1.0$ [cm cm$^{-3}$].
Figure 41 shows the experimental data from day 40 (Figure 39) modelled assuming that the roots occupied the total pot volume and using constant values of $D$ that range from $10 \times 10^{-9}$ to $1 \times 10^{-9}$ [m$^2$ s$^{-1}$] for $L_v = 2.0$ [cm cm$^{-3}$]. Figure 41(a), (b), (c) and (d) show that when $D$ was greater than or equal to $3 \times 10^{-9}$ [m$^2$ s$^{-1}$], the model showed very little evidence that $\Delta \tau$ becomes appreciable at large $E$. The model in Figure 41(a) and (b), where $D = 2 \times 10^{-9}$ and $1 \times 10^{-9}$ [m$^2$ s$^{-1}$], suggests that $\Delta \tau$ is becoming appreciable: however, as in Figure 40, the shape of the model does not match the experimental data, especially the acceleration of $B$ with $E$. 
Figure 41 Second replicate of undisturbed clay-loam soil: model of experimental data from day 40 (from Figure 39). The data was modelled assuming the roots had explored the total pot volume and using a range of constant values of $D$ [m$^2$ s$^{-1}$] for $L_r = 2.0$ [cm cm$^{-3}$].
Figure 42 shows the experimental data from day 40 (Figure 39) modelled assuming that the roots occupied the total pot volume and using constant values of $D$ that ranged from $10 \times 10^{-9}$ to $1 \times 10^{-9}$ [m$^2$ s$^{-1}$] and values of $L_v$ that range from 4.0 to 0.5 [cm cm$^{-3}$]. The values of $L_v$ used in Figure 42 were mostly greater than or less than those used in Figure 41 and Figure 40, thereby the model could be investigated with low $D$ and high $L_v$ and vice versa. These plots represent combinations of the upper and lower values of both $D$ and $L_v$; Figure 42(a) uses $D = 1 \times 10^{-9}$ [m$^2$ s$^{-1}$] and $L_v = 4.0$ [cm cm$^{-3}$] and Figure 42(f) uses $D = 10 \times 10^{-9}$ [m$^2$ s$^{-1}$] and $L_v = 0.5$ [cm cm$^{-3}$]. As in Figure 40 and Figure 41, none of the plots in Figure 42 show complete agreement between the experimental data and the model.
Figure 42 Second replicate of undisturbed clay-loam soil: model of experimental data from day 40 (from Figure 39). The data was modelled assuming the roots had explored the total pot volume and using a range of constant values of $D$ [m$^2$ s$^{-1}$] and $L_v$ [cm cm$^{-3}$].
Given that none of the modelling attempts which assumed that the roots occupied the total soil volume agreed with the data and that the roots for the second replicate were not seen at the base of the core, the data was then modelled assuming that the roots occupied the top 75% or 50% of the total pot volume. Figure 43 shows the experimental data from day 40 (Figure 39) modelled assuming that the roots occupied the top 75% of the total pot volume and using constant values of $D$ that range from $6 \times 10^{-9}$ to $3 \times 10^{-9}$ $[m^2 \text{s}^{-1}]$ for $L_v = 2.0$ $[\text{cm cm}^{-3}]$. None of the modelling attempts in Figure 43 show agreement between the experimental data and the model; the acceleration of $B$ with $E$, seen in the experiment, was not predicted by the model.

Figure 43 Second replicate of undisturbed clay-loam soil: model of experimental data from day 40 (from Figure 39). The data was modelled assuming the roots had explored 75% of the pot volume and using a range of constant values of $D$ $[m^2 \text{s}^{-1}]$ for $L_v = 2.0$ $[\text{cm cm}^{-3}]$. 
Figure 44 shows the experimental data from day 40 (Figure 39) modelled assuming that the roots occupied the top 75% of the total pot volume and using constant values of $D$ that range from $30 \times 10^{-9}$ to $7 \times 10^{-9}$ [m$^2$ s$^{-1}$] for $L_v = 0.5$ and 1.0 [cm cm$^{-3}$]. None of the modelling attempts match the experimental data.

Figure 44 Second replicate of undisturbed clay-loam soil: model of experimental data from day 40 (from Figure 39). The data was modelled assuming the roots had explored 75% of the pot volume and using a range of constant values of $D$ [m$^2$ s$^{-1}$] for $L_v = 1.0$ and 0.5 [cm cm$^{-3}$].

Figure 45 shows the experimental data from day 40 (Figure 39) modelled assuming that the roots occupied the top 50% of the total pot volume and using constant values of $D$ that range from $50 \times 10^{-9}$ to $20 \times 10^{-9}$ [m$^2$ s$^{-1}$] for $L_v = 0.5$ and 0.75 [cm cm$^{-3}$]. None of the
modelling attempts, even qualitatively, agree with the experimental data; in every case the slope of the rising $E$ phase, for the model, was steeper than that of the experimental data, thereby indicating that, in the model, $\Delta \tau$ was becoming appreciable with increasing $E$; this is because the model assumes the same $R_{Plant}$ determined from the linear slope of increasing $E$ from the experiment, and if $\Delta \tau$ was negligible in the model, then the modelled line of increasing $E$ would be parallel to (in the case that was error in the estimate of the intercept or the soil water retention) or on top of the experimental line of increasing $E$ (see discussion of Figure 33 in section 5.1). However, if $\Delta \tau$ in the experiment was increasing in proportion to increasing $E$, then the slope of the line of increasing $E$ will over estimate of $R_{Plant}$; therefore $R_{Plant}$ is only an estimate. Importantly, the acceleration of $B$ with large $E$, evident in the experimental data, was not evident in the modelled data.
Figure 45 Second replicate of undisturbed clay-loam soil: model of experimental data from day 40 (from Figure 39). The data was modelled assuming the roots had explored 50% of the pot volume and using a range of constant values of \( D [\text{m}^2 \text{s}^{-1}] \) for \( L_v = 0.75 \) and \( 0.5 [\text{cm} \text{cm}^{-3}] \).
Figure 46 shows the experimental data from day 40 (Figure 39) modelled assuming that the roots occupy the top 50% of the total pot volume and using constant values of $D$ that range from $30 \times 10^{-9}$ to $10 \times 10^{-9}$ [m$^2$ s$^{-1}$] for $L_r = 1.0$ and 2.0 [cm cm$^{-3}$]. The results were similar to Figure 45, where none of the modelling attempts showed agreement with the experimental data; in all of the plots, the modelled plot of increasing $E$ was steeper than the experimental data, and the acceleration of $B$ with large $E$, evident in the experimental data, was not evident in the modelled data.

Figure 46 Second replicate of undisturbed clay-loam soil: model of experimental data from day 40 (from Figure 39). The data was modelled assuming the roots had explored 50% of the pot volume and using a range of constant values of $D$ [m$^2$ s$^{-1}$] for $L_r = 1.0$ and 2.0 [cm cm$^{-3}$].
Figure 47 shows the experimental data from day 40 (Figure 39) modelled assuming that the roots occupy the top 50% of the total pot volume and using constant values of $D$ that range from $9 \times 10^{-9}$ to $5 \times 10^{-9}$ [m$^2$ s$^{-1}$] for $L_v = 2.0$ [cm cm$^{-3}$]. In each of the plots, the modelled slope of increasing $E$ was steeper than that of the experimental data and the acceleration of $B$ with large $E$, evident in the experimental data, was not evident in the modelled data.

In summary, the key finding from the experimental and modelled data of the undisturbed clay-loam soil was that the acceleration of $B$ with large $E$, evident in the experimental
data, was not evident in the modelled data. None of the modelling attempts have shown close agreement with the experimental data. When it was assumed that the roots were occupying the total pot or the top 75% of the pot, modelling with various combinations of $L_v$ and $D$ could not match the experimental data. Closer agreement between the model and the experimental data resulted when it was assumed that the roots were confined to the top 50% of the pot; however none of the modelling attempts fitted the experimental data over the whole range of the rising and falling $E$ phases.

The model assumes that the distribution of water and roots in the soil is homogenous; this assumption is unlikely to be valid in the undisturbed clay-loam soil, and may contribute to the non-agreement between the experimental data and the model. The following two sections report results from plant water uptake experiments using repacked soil compared to the model; where it was expected that the assumption that the roots are homogenously distributed will be more valid, and thereby enable testing of the hypothesis that the soil was the main resistance to the extraction of water by the plant roots, due to the combined effects of low $D(\theta)$ and low $L_v$ at low $\theta$.

5.3.2 Repacked clay-loam

Three replicates of the repacked clay-loam soil were used for measurements of $B$ and $E$. The most reliable measurements were made on the third replicate and these were analysed in detail. The measurements made on the first and second replicates are included in Appendix 4.

The data from days 22 to 32 (shown in Appendix 4) were essentially linear, for rising and falling $E$ phases, with $\Delta \tau$ apparently negligibly small. The data from days 32, 35, 37 and 39 is plotted in Figure 48. The slope and intercept of the linear parts of each increasing and decreasing plot of $B$ as a function of $E$, shown in Appendix 4 and Figure 48, are presented in Table 7.
Table 7 Third replicate of repacked clay-loam soil: slope, intercept and leaf area for each of the B and E experiments shown in Appendix 4 and Figure 48. Note that the plant was re-watered back to 10 kPa suction weight after the experiment on day 37. (up) and (down) denotes rising and falling E phases respectively.

<table>
<thead>
<tr>
<th>Age (days from sowing)</th>
<th>Intercept (up) [kPa]</th>
<th>Slope (up) [kPa s µg⁻¹]</th>
<th>Intercept (down) [kPa]</th>
<th>Slope (down) [kPa s µg⁻¹]</th>
<th>Leaf Area [cm²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>9</td>
<td>2.7</td>
<td>67</td>
<td>2.1</td>
<td>17.9</td>
</tr>
<tr>
<td>24</td>
<td>35</td>
<td>2.2</td>
<td>39</td>
<td>2.2</td>
<td>23.7</td>
</tr>
<tr>
<td>26</td>
<td>24</td>
<td>2.0</td>
<td>63</td>
<td>1.8</td>
<td>29.0</td>
</tr>
<tr>
<td>28</td>
<td>58</td>
<td>1.7</td>
<td>57</td>
<td>1.7</td>
<td>39.4</td>
</tr>
<tr>
<td>30</td>
<td>27</td>
<td>1.5</td>
<td>94</td>
<td>1.3</td>
<td>49.7</td>
</tr>
<tr>
<td>32</td>
<td>73</td>
<td>1.5</td>
<td>181</td>
<td>1.3</td>
<td>63.1</td>
</tr>
<tr>
<td>35</td>
<td>288</td>
<td>2.1</td>
<td>534</td>
<td>2.7</td>
<td>82.4</td>
</tr>
<tr>
<td>37</td>
<td>870</td>
<td>2.3</td>
<td>975</td>
<td>2.8</td>
<td>94.8</td>
</tr>
<tr>
<td>39</td>
<td>-4</td>
<td>1.4</td>
<td>19</td>
<td>1.3</td>
<td>112.7</td>
</tr>
</tbody>
</table>

After the experiment on day 39, the soil was washed from the roots for determination of total root length and diameter (Table 8): nearly half of the total root length was in the top 5 cm and for the bottom 15 cm, the root length distribution with depth was approximately uniform.

Table 8 Third replicate plant grown in repacked clay-loam soil: measurements of total root length and diameter.

<table>
<thead>
<tr>
<th>Depth [cm]</th>
<th>Average root diameter [mm]</th>
<th>Total root length [m]</th>
<th>Root length density [cm cm⁻³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-5</td>
<td>0.27</td>
<td>30.8</td>
<td>10.8</td>
</tr>
<tr>
<td>5-10</td>
<td>0.30</td>
<td>13.5</td>
<td>4.8</td>
</tr>
<tr>
<td>10-15</td>
<td>0.31</td>
<td>12.0</td>
<td>4.2</td>
</tr>
<tr>
<td>15-20</td>
<td>0.42</td>
<td>11.9</td>
<td>4.2</td>
</tr>
</tbody>
</table>

The re-watering of the plant back to the 10 kPa suction weight on day 37 may have contributed to the greater total root length in the first 5 cm (Table 8). The roots in the first 5 cm may have proliferated upon re-watering of the plant; the same speculation can be made with the roots growing in the bottom 15 cm of the pot. For the purpose of modelling the plant water uptake, the measurements of total root length provide an upper bound on what the total root length might have been at a particular point in time.
That the data on days 32 and 39 (Figure 48), during the rising and falling $E$ phases, was relatively linear, indicated that $\Delta \tau$ was negligibly small. The intercept for the rising $E$ phase on day 35 (Figure 48 and Table 7) was approximately 215 kPa greater than that from day 32, showing that the bulk soil had started to dry appreciably. On day 35 $B$ was a linear function of $E$ for the first four evaporative demand settings; when the plant was exposed to the fifth evaporative demand setting, for the first five to ten minutes, the relationship between $B$ and $E$ was linear at which point $B$ then increased non-linearly with $E$; indicating that $\Delta \tau$ was starting to become appreciable. The final evaporative demand setting was maintained for 2 h and 20 min and each point on the non-linear part of the data from day 35 (Figure 48) represents a measurement made every five min. When the evaporative demand was decreased the return slope was similar to that from the rising $E$ phase, however the intercept increased 246 kPa; indicating that the bulk soil had dried by that amount.

Figure 48 Third replicate of repacked clay-loam soil: $B$ as a function of $E$, experiment and model for days 32 to 39. Model in both (a) and (b) assume 10% of roots are taking up water, (a) uses variable $D(\theta)$ (eq. 55) and constant $R_{\text{plant}}$ of 2.1, for day 35, and 1.8, for day 37. (b) uses constant $D$ and
constant $R_{plant}$ of 1.6, for day 35, and 1.5, for day 37. Black and red lines denote rising and falling $E$ phases respectively.

At the highest evaporative demand setting, for day 35 in Figure 48, $E$ actually started to decrease while $B$ was still increasing, where presumably the stomates were closing. Figure 49 shows $B$, $E$ and the stomatal conductance, $g$, plotted with cumulative time (for day 35). It is apparent from Figure 49 that the stomates continued to open as $E$ and $g$ steadily increased, then the horizontal bar in Figure 48 indicates that $E$ and $g$ plateau before starting to decrease, where presumably the stomates were closing; while at the same time $B$ was still increasing. From Figure 48 and Figure 49 it is clear that $B$ continued to increase independently of $E$ and $g$ and that therefore; (1) the resistance to flow within the system was increasing (ratio of $B$ and $E$) and (2) if the combined effects of low $D(\theta)$ and low $L_v$ at low $\theta$ are the main resistance to the extraction of water by the roots, as hypothesized, then the model should agree with the data on day 35.

![Graph showing balancing pressure, transpiration rate, and stomatal conductance](image)

**Figure 49** Third replicate of repacked clay-loam soil, experimental data from day 35: balancing pressure, $B$, transpiration rate, $E$, and stomatal conductance, $g$, plotted with cumulative time.
Horizontal lines above plots of $E$ and $g$ show the plateau and subsequent decrease in the two parameters, while $B$ continues to increase independent of $E$.

In contrast to the hypothetical model shown in Figure 33 (section 5.1), which showed $B$ going to singularity at high $E$, $B$ on day 35 (Figure 48) does not reach singularity, because, as Figure 49 shows, $E$ becomes constant before falling; whereas in Figure 33 $E$ is continually increased at the rate of 1 [$\mu$g s$^{-1}$] per min.

The response by the plant to close the stomates when the pressure drop across the plant and soil is high and seemingly increasing without bound is possibly due to the shoot receiving a signal from the roots that are directly experiencing the drying soil. It is reasonable to assume that signals are not originating from the leaf because this experimental technique essentially clamps the pressure of the xylem sap at zero; the water status of the shoot remains constant, equal to atmospheric pressure throughout the experiment. Passioura & Gardner (1990), using the same experimental technique, found that for plants growing in drying soil, the relative leaf expansion rate decreased as the soil dried and concluded that the roots were sensing the drying soil and sending signals to the shoot.

Attempts to model the experimental data from days 35 and 37 are shown in Figure 48; in both plots, the intercept (Table 7) for the rising $E$ phase was used to determine the initial soil water content for the plant water uptake model using the fitted van Genuchten (1980) parameters shown in Table 5. The initial soil water content was assumed uniform with depth. The main difference between the two modelling attempts, shown in Figure 48, is the $D(\theta)$ functions used: Figure 48(a) used a quadratic $D(\theta)$, eq. 55, measured on the clay-loam repacked to 1.3 [g cm$^{-3}$] (shown in Figure 15, section 3.3.3), and Figure 48(b) used a constant $D$ of $2\times10^{-9}$ and $8\times10^{-9}$ [m$^2$ s$^{-1}$] for days 35 and 37 respectively. The constant value of $D$ was chosen to enable the best fit of the model to the data, and in both cases was less than the measured minimum $D$, for the clay-loam repacked to 1.3 [g cm$^{-3}$], of $10\times10^{-9}$ (Table 2, section 3.4). The model significantly overestimated $B$ when the experimental data from day 37 was modelled using $D$ of $2\times10^{-9}$ [m$^2$ s$^{-1}$].
The other difference between the two modelling attempts, shown in Figure 48, was the \( R_{\text{Plant}} \) used. In Figure 48(a), \( R_{\text{Plant}} \) of 2.1 and 1.8 [kPa s \( \mu \text{g}^{-1} \)] was used for days 35 and 37 respectively. The \( R_{\text{Plant}} \) of 2.1 [kPa s \( \mu \text{g}^{-1} \)] was the same as that estimated from the experiment on day 35. However the closest agreement between experiment and model for day 37, when using \( D(\theta) \) in eq. 55, was achieved using \( R_{\text{Plant}} \) of 1.8 [kPa s \( \mu \text{g}^{-1} \)]; less than the \( R_{\text{Plant}} \) of 2.3 [kPa s \( \mu \text{g}^{-1} \)] (Table 7) estimated from the experiment. In Figure 48(b) the model assumed that the \( R_{\text{Plant}} \) was 1.6 and 1.5 [kPa s \( \mu \text{g}^{-1} \)] for days 35 and 37 respectively; less than the \( R_{\text{Plant}} \) of 2.1 (day 35) and 2.3 (day 37) estimated from the experiment. Figure 33 and the discussion in section 5.1, demonstrate that the slope of increasing \( E \) can still appear linear while \( \Delta \tau \) is appreciable, in which case the slope of the line of increasing \( E \), will overestimate \( R_{\text{Plant}} \).

The model using variable \( D(\theta) \), from eq. 55, (Figure 48(a)) underestimated the steep part of the experimental data, on days 35 and 37, and did not agree with the experimental data. This would suggest that something other than low \( D(\theta) \) and low \( L_v \) at low \( \theta \), is causing the acceleration of \( B \) with \( E \).

The model of day 35, using a constant \( D \) of 2x10\(^{-9} \), (Figure 48(b)) matched the non-linear part of the experimental data, where \( B \) accelerated rapidly with \( E \). However the drying of the bulk soil was underestimated by the model, because the intercept of the model is less than that of the experiment. The model of day 37 did not agree with the experimental data. The maximum \( B \) at the greatest \( E \) was not reached and the slope of the falling \( E \) phase was linear and mostly underestimated \( B \) when \( E \) was decreasing.

The data for day 35 (Table 7 and Figure 48) indicates that the slope of \( B(E) \) for the falling \( E \) phase (2.7 [kPa s \( \mu \text{g}^{-1} \)]) was greater than for the rising \( E \) phase (2.1 [kPa s \( \mu \text{g}^{-1} \)]), suggesting that the rapid rise in \( B \) when \( E \) was high may have caused the resistance to flow within the system to increase. The apparently increased constant resistance, for the falling \( E \) phase, may be due to a proportional fall in \( \Delta \tau \) with \( E \). However, the constant slope during the falling \( E \) phase suggests that the radial flow of water through the soil to the plant roots generated only minor gradients in soil suction. That is the flow of water towards the roots was not meeting a large hydraulic resistance.
Figure 50(a) shows the data from day 35 modelled using a constant $D$ of $2\times 10^{-9}$ [m$^2$ s$^{-1}$] (as in Figure 48(b)) and variable $R_{plant}$ of 1.6 and 2.0 [kPa s µg$^{-1}$] for rising and falling $E$ phases respectively. Despite using variable resistances that were lower than that estimated from the experiment, the model was in closer agreement with the experimental data than the modelling attempt shown in Figure 48(b). The apparently increased constant resistance, when $E$ was falling, may be due to an interfacial resistance developing when $E$ was high.

Figure 50(b) shows the model output with time of $B$, the suction at the root surface and in the bulk soil from the model simulation of day 35 shown in Figure 50(a). The modelled suction at the root surface fell significantly after 250 min, which corresponded to when the evaporative demand was decreased from the maximum setting to the second maximum, and the measured $E$ decreased from 460 to 293 [µg s$^{-1}$]. The modelled suction in the bulk soil increased 60 kPa for the total duration of the experiment. However the difference between the intercepts of the rising and falling $E$ phases, shown in Table 7, is 246 kPa; the model underestimated the apparent drying of the bulk soil, which may have been due to the uneven distribution of roots with depth. The model assumed that the root distribution was uniform with depth, but Table 8 provides evidence that the roots were concentrated in the top 5 cm of the core.
Figure 50 Third replicate grown in repacked clay-loam soil: (a) shows experiment and model of day 35, using constant $D=2 \times 10^{-9}$ [m$^2$ s$^{-1}$] and variable $R_{\text{fuit}}$ of 1.6 and 2.0 [kPa s $\mu$g$^{-1}$] for rising and falling $E$ phases respectively. Black and red lines denote rising and falling $E$ phases respectively. (b) shows modelled $B$, soil matric suction at root surface and in the bulk soil as a function of time.

5.3.3 Repacked sand

Measurements of $B$ and $E$ were made on two replicates of repacked sand. The most reliable measurements were made on the second replicate and these were analysed in detail. The measurements of $B$ and $E$ made on the first replicate of repacked sand are shown in Appendix 4.

Measurements of $B$ and $E$ were made on the second replicate of repacked sand on six different days; the slope and intercept for the linear part of each line of the rising and falling $E$ phases are shown in Table 9. The plots of experimental data from days 21 to 29 were linear for both the rising and falling $E$ phases and are shown in Appendix 4. Figure 51 shows the data from days 31 and 33.
Table 9 Second replicate of repacked sand: slope and intercept for each of the B and E experiments shown in Appendix 4 (days 21 to 29) and Figure 51 (days 31 and 33). (up) and (down) denotes rising and falling E phases respectively.

<table>
<thead>
<tr>
<th>Age (days from sowing)</th>
<th>Intercept (up) [kPa]</th>
<th>Slope (up) [kPa s µg⁻¹]</th>
<th>Intercept (down) [kPa]</th>
<th>Slope (down) [kPa s µg⁻¹]</th>
<th>Leaf area [cm²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>-42</td>
<td>3.0</td>
<td>1</td>
<td>2.7</td>
<td>19.2</td>
</tr>
<tr>
<td>24</td>
<td>1</td>
<td>2.0</td>
<td>25</td>
<td>1.9</td>
<td>27.4</td>
</tr>
<tr>
<td>27</td>
<td>-25</td>
<td>1.8</td>
<td>24</td>
<td>1.6</td>
<td>41.8</td>
</tr>
<tr>
<td>29</td>
<td>-9</td>
<td>1.6</td>
<td>28</td>
<td>1.5</td>
<td>55.3</td>
</tr>
<tr>
<td>31</td>
<td>7</td>
<td>1.5</td>
<td>14</td>
<td>2.9</td>
<td>68.1</td>
</tr>
<tr>
<td>33</td>
<td>12</td>
<td>2.8</td>
<td>37</td>
<td>4.5</td>
<td>79.7</td>
</tr>
</tbody>
</table>

The experiments conducted on days 21 to 29 exhibited negative or close to zero intercepts for the rising E phase (Table 9). The literal interpretation of a negative intercept, using eq. 51, is that the soil water pressure was positive and therefore the soil was at a water content equal to saturation. It was clear that this was not the case and that the negative intercepts for the plots during the rising E phase indicate that the soil was still quite wet relative to the plant. The intercepts of the rising E phase on days 31 and 33 (Table 9) were positive and close to the initial suction of 10 kPa indicating that the suction in the bulk soil was still quite high.

Table 9 also shows that the return intercepts for the falling E phase on days 21 to 29 were all positive. Days 24, 27 and 29, were very similar and were within 17.5 kPa of the initial suction of 10 kPa. The intercepts of the falling E phase on days 31 and 33 were 7.5 and 25.3 kPa greater, respectively, than those for the rising E phase.

The average root diameter, total root length and root length density in 5 cm depth increments for the plant grown in repacked sand is presented in Table 10. The total root length for the sand (36.2 m) was 53% of the repacked clay-loam (68.3 m). In contrast to the repacked clay-loam soil, the distribution of roots in the sand was approximately uniform with depth for the top 15 cm and the total root length in the bottom 5 cm is 12 to 50% less than that of the other 5 cm depth increments. Also, unlike the repacked clay-loam, the repacked sand was not re-watered and the total root length of the sand was measured the day after the last attempted plant water uptake experiment, on day 35.
Table 10 Second replicate of repacked sand: measurements of total root length and diameter.

<table>
<thead>
<tr>
<th>Depth [cm]</th>
<th>Average root diameter [mm]</th>
<th>Total root length [m]</th>
<th>Root length density [cm cm⁻³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-5</td>
<td>0.27</td>
<td>8.0</td>
<td>2.9</td>
</tr>
<tr>
<td>5-10</td>
<td>0.27</td>
<td>9.4</td>
<td>3.4</td>
</tr>
<tr>
<td>10-15</td>
<td>0.33</td>
<td>12.2</td>
<td>4.4</td>
</tr>
<tr>
<td>15-20</td>
<td>0.35</td>
<td>6.5</td>
<td>2.3</td>
</tr>
</tbody>
</table>

The experimental data from days 31 and 33 in Figure 51 were modelled assuming that the \( R_{Plant} \) was 1.5 and 2.8 [kPa s µg⁻¹] respectively, which was the same as the estimated values of \( R_{Plant} \) for the rising \( E \) phase, shown in Table 9. The intercept (Table 9) for the rising \( E \) phase was used to determine the initial soil water content for the plant water uptake model using the fitted van Genuchten (1980) parameters in Table 5. The initial soil water content and the root length density were assumed uniform with depth. The model assumed that 10% of the total measured root length, in Table 10, was extracting water.
Figure 51 Second replicate of repacked sand: $B$ as a function of $E$, experiment and model, for days 31 and 33. Model in all plots assumes 10% of roots were taking up water and $R_{\text{Plant}}$ is 1.5 and 2.8 [kPa s $\mu$g$^{-1}$] for days 31 and 33 respectively, (a) and (c) use variable $D(\theta)$ (eq. 56), (b) and (d) use constant $D$. Black and red lines denote rising and falling $E$ phases respectively.

Figure 51(a) and (c), were modelled using the quadratic $D(\theta)$ shown in eq. 56 (shown in Figure 16 as quadratic number two, in section 3.3.4), and Figure 51(b) and (d) were modelled using a constant $D$ of 1x10$^{-9}$ [m$^2$ s$^{-1}$]; chosen to enable the closest fit of the
model to the experimental data. The constant $D$ of $1 \times 10^{-9}$ [m$^2$ s$^{-1}$] was within the range of the quadratic $D(\theta)$ functions from the evaporation experiments, but lower than both the exponential $D(\theta)$ functions, also from the evaporation experiments, and the $D(\theta)$ functions derived from the outflow experiment (Figure 20, section 3.4).

On day 31 (Figure 51), the first four points of $B$ as a function of $E$ were linear and represent the mean of measurements made over 15 to 20 min. The fifth and subsequent points until $B$ begins to decrease, were made at the highest evaporative demand setting. This setting was maintained for 2.5 h and each point on the non-linear part of day 31 (Figure 51) represents an average measurement made over 5 min. The data from day 31 clearly show that $B$ accelerates rapidly with $E$, suggesting that $\Delta \tau$ increased appreciably with $E$. However the model on day 31 (shown in Figure 51(a)) does not agree with the experimental data, when clearly $B$ is accelerating and eventually increasing independently with $E$. This discrepancy is evidence that the soil hydraulic properties are not generating a large resistance to the extraction of water by the roots from the soil.

The experimental data from day 31 (Figure 51) also shows that $E$ started to decrease while $B$ continued to increase. This behaviour, where the plant $E$ decreased while the evaporative demand remained the same, was very similar to that of the plant grown in the repacked clay-loam soil (Figure 48); where it was postulated that the roots sensed the drying soil and sent signals to the leaves resulting in the stomates closing and $E$ decreasing. The rise, then plateau and fall of both $E$ and $g$ (for day 31) is shown in Figure 52, where $B$, $E$ and $g$ are plotted with cumulative time. That Figure 52 shows that $B$ continues to increase independently of $E$ and $g$, indicates that there was an increasing resistance to flow through the soil and plant, which could have developed in the soil, at the interface between the soil and the root or within the plant.
Figure 52 Second replicate of repacked sand, experimental data from day 31: balancing pressure, $B$, transpiration rate, $E$, and stomatal conductance, $g$, plotted with cumulative time. Horizontal lines above plots of $E$ and $g$ show the plateau and subsequent decrease in the two parameters, while $B$ continues to increase independent of $E$.

The model of $B$ as a function of $E$ for days 31 and 33 (Figure 51(a) and (c)) underestimated $B$ when modelled with the variable $D(\theta)$ shown in eq. 56; thereby disproving the hypothesis that the soil hydraulic properties are the main resistance to the uptake of water by plant roots due to low $D(\theta)$ and low $L_v$ at low $\theta$. However, closer agreement between the experiment and the model, for the rising $E$ phase only, on day 31 was attained by using a constant $D$ of $1 \times 10^{-9} \text{ m}^2 \text{s}^{-1}$ (Figure 51(b)). Although, the model did not agree with the data from day 33 when the same value of $D$, $1 \times 10^{-9} \text{ m}^2 \text{s}^{-1}$ (Figure 51(d), was used, thereby suggesting that low $D$ alone is not the main resistance to the uptake of water by the plant roots.

The slope of the falling $E$ phase on day 31 was nearly twice that of the rising $E$ phase and the intercept is similar for both the rising and falling $E$ phases (Table 9). Both slopes were linear, indicating that the “extra” resistance apparent when $E$ was falling was constant. This can be interpreted by considering that if the bulk soil had dried significantly, then the intercept for the falling $E$ phase would be greater than that for the rising $E$ phase, and
if the resistance to flow within the system remained the same, the slopes for the rising and falling $E$ phases would be the same. In contrast, if the bulk soil did not dry, and if the resistance to flow within the system remained the same, then the plot of the falling $E$ phase would return back along the plot of the rising $E$ phase, as the model did in Figure 51(b).

The greater slope for the falling $E$ phase, on day 31 (Figure 51(a) and (b) and Table 9), may have been caused by hysteresis in the soil close to the root. If there was significant drying of the soil close to the root when $E$ was high, as indicated in Figure 51, then soil water hysteresis may result in the soil close to the root surface re-wetting to a lesser $\theta$ at a given suction. This would create a lower $D$ at the given suction than previous and would require a greater suction gradient and hydraulic conductivity close to the root to maintain a given $E$, which may contribute to the increased apparent resistance when $E$ was falling (greater return slope). However this apparently greater resistance was constant and an extra resistance within the soil would tend to be variable, given that the soil hydraulic properties are typically non-linear.

To investigate the hypothesis that the hysteresis in the sand prevented the soil close to the root from re-wetting, the drying and wetting soil water retention curves for the sand were measured (Figure 53) to 25 kPa suction. Figure 53 shows that the hysteresis in the sand at 20 kPa suction was less than 5%, but was close to 15% for 15 and 10 kPa suctions.
Figure 53 Repacked sand: drying and re-wetting soil water retention curve. Plot shows two hysteresis loops: 8 and 25 kPa suction.

Figure 54 shows the model output of the suction at the root surface, bulk soil suction and the $B$ with time from the model simulation of day 31, using a constant $D$ of $1 \times 10^{-9}$ [m$^2$ s$^{-1}$] (from Figure 51(b)). There was negligible change in the suction in the bulk soil, which agrees with Table 9, where the intercept increased from 7 to 14 kPa. However the suction at the root surface increased significantly from about 200 min and the rate of change with time was rapid before a maximum of 759 kPa was attained at 265 min. At that point the evaporative demand was reduced and the suction at the root surface fell sharply. The suction at the root surface started to equilibrate at approximately 60 kPa after 290 min and then fell again and equilibrated at 40 kPa after 320 min when the evaporative demand was reduced. At 350 min, the root suction equilibrated at 20 kPa after the evaporative demand was reduced and finally the root suction equilibrated at 15 kPa after 375 min when the evaporative demand was at the original setting.
The small hysteresis at suctions drier than 20 kPa suction (Figure 53) and the model output of suction at the root surface with time (Figure 54(b)), which for the most part was drier than 20 kPa, indicate that the two fold increase in resistance when $E$ was falling, on day 31 in Figure 51, was unlikely to be due to hysteresis in the soil hydraulic properties alone. The slope of $B(E)$ when $E$ was falling was remarkably linear, indicating that a step-change increase in the resistance to flow within the system had occurred. The discussion of the hypothetical model in Figure 33 (section 5.1) showed that if $\Delta \tau$ were decreasing in proportion with decreasing $E$, then the slope of $E$ would be greater and also linear.

The closest agreement between experiment and model, for day 31, was achieved by invoking an “extra” resistance, nearly equal to that of the plant, when the evaporative demand was decreased. Figure 55 shows that the model was able to simulate the experimental data when assuming that 10% of the roots were extracting water, that $D$ was constant at $1 \times 10^{-9}$ [m$^2$ s$^{-1}$] and that $R_{Plant}$ was variable at 1.5 and 2.9 [kPa s $\mu$g$^{-1}$] for the rising and falling $E$ phases respectively. That an “extra” resistance, nearly equal to the
apparent hydraulic resistance of the plant, was required for the model to agree with the experimental data demonstrates that low $D(\theta)$ and low $L_v$ alone were not generating a large resistance to the extraction of water by the plant roots.

![Graph](image)

**Figure 55** Second replicate of repacked sand: $B$ as a function of $E$, experiment and model, for day 31, also shown in Figure 51. Data was modelled using constant $D = 1 \times 10^{-9}$ [m$^2$ s$^{-1}$] and variable $R_{Plant}$ of 1.5 and 2.9 [kPa s $\mu$g$^{-1}$] for increasing and decreasing $E$ respectively. Black and red lines denote rising and falling $E$ phases respectively.

The step-change increase in resistance of 1.4 [kPa s $\mu$g$^{-1}$] invoked when $E$ was decreasing (Figure 55) may be due to an interfacial resistance developing at the root surface when the evaporative demand was high. The interfacial resistance may have been due to root shrinkage when the soil water content at the root surface was low and the evaporative demand was high. However the root diameter would presumably recover when the evaporative demand was decreased and the soil at the root surface was re-wet. This would result in a decreasing non-linear resistance, in contrast to the step-change in resistance shown in Figure 55. If the hydraulic continuity between the root surface and the soil was
reduced, due to root shrinkage or some other interfacial process, and was not restored when the evaporative demand was decreased, then the increased resistance could be constant.

The extra resistance invoked in Figure 55 could be attributed in part to a build up of solutes in the rhizosphere not being accounted for in the model. The sandy soil used in this experiment may have exacerbated the possible build up of solutes in the rhizosphere (Stirzaker & Passioura 1996). However Stirzaker & Passioura (1996) watered their plants with a nutrient solution of 70 kPa osmotic pressure compared to 26 kPa in this study, so any build up of solute was less likely to occur here or would be of a smaller magnitude.

The model simulation, when $R_{Plant}$ was constant, underestimates the return intercept on both days 31 and 33 (Figure 51). This disagreement may represent the osmotic pressure in the rhizosphere and in the bulk soil that was not simulated by the model. Also the return intercept on day 31 was greater than the increasing intercept on day 33 (Table 9). This could be due to solutes in the rhizosphere on day 31 back diffusing away from the root out into the bulk soil solution such that by day 33 the apparent total soil water potential in the bulk soil was less negative.

The negligible drying of the bulk soil, evident in both the experimental and modelled data in Figure 51 and Figure 54, was similar to the field experiment of Carbon (1973), where the bulk soil did not dry more than 100 kPa in any of his treatments. Carbon (1973) found that the leaf water potential of sorghum plants grown on coarse sandy soil exhibited rapid diurnal change from no plant water stress at dawn to large negative potentials in the mid-afternoon. He concluded that despite the negligible drying of the bulk soil the large negative leaf water potentials were due to the unavailability of the soil water to meet the plant demand.

5.4 Conclusion

The aim of this chapter was to explore the hypothesis that the soil is the main resistance to the extraction of water by the plant roots, owing to a combination of low $L_v$ (or $r_b$, see eq. 2) at low $D(\theta)$ at low $\theta$. This hypothesis was tested by comparing experiments that
simultaneously measure $B$ and $E$, from plants grown in pots of undisturbed clay-loam field soil, repacked clay-loam and repacked sand, with the single root model.

For all three soil types, during the rising $E$ phase, $B(E)$ was linear at low to moderate $E$, with a constant slope that, when $\Delta \tau$ was negligibly small, represented the hydraulic resistance of the plant, $R_{\text{Plant}}$. A model of a hypothetical experiment was used to demonstrate that if $\Delta \tau$ increased in proportion with increasing $E$, then the slope of increasing $E$ would still be linear and that therefore $R_{\text{Plant}}$ would be overestimated. At high $E$, $B$ accelerated rapidly with $E$. During the falling $E$ phase, for the repacked clay-loam and the repacked sand, $B(E)$ was essentially linear over the whole range of $E$. This was in contrast to the undisturbed clay-loam, where $B$ was often curved downwards with decreasing $E$, thereby suggesting that substantial suction gradients may have developed close to the root during the rising $E$ phase, and may have collapsed during the falling phase.

For the repacked clay-loam and the repacked sand, the model could match the data during the rising phase of $E$, if it was assumed that only 10% of the roots were taking up water and if $D$ was constant and low. However the model could not match the data during the falling $E$ phase, unless it was assumed that there had been a significant rise in the hydraulic resistance of the plant, or more likely, that an additional interfacial resistance had developed when $E$ was high and $B$ was rapidly increasing. The postulated interfacial resistance was about the same size as that within the plant, for the repacked sand, and about a quarter of that within the plant for the repacked clay-loam.

That the slope of $B(E)$ during the falling $E$ phase, for the repacked clay-loam and the repacked sand, was essentially constant suggests that the radial flow of water through the soil to the plant roots generated only minor gradients in soil suction and that low $D(\theta)$ and low $L_v$ alone are not generating a large resistance to flow in the soil. The results on the repacked soil suggest that an additional constant resistance developed when $E$ was large and $B$ was rapidly increasing. That the additional resistance was constant, was in contrast to other findings of interfacial resistance, where the resistance was effectively a function of the soil water content (Bristow, Campbell & Calissendorff 1984; Herkelrath,
Miller & Gardner 1977b). The additional resistance encountered in this study may have been caused by the roots shrinking, when $E$ was large and not rehydrating when $E$ was reduced. This would mean that the roots did not recover, or only partially recovered, when the evaporative demand was reduced. Such behaviour would be consistent with the experience of Faiz & Weatherley (1982), who found that under high transpiration, squeezing or vibrating the soil containing sunflower roots, temporarily reduced the plant water stress.

The hypothesis that root shrinkage occurs at high $E$ and that the root diameter only partly recovers when $E$ is decreased, which results in a step-change increase in the apparent resistance to flow within the system, could be tested by combining the experimental technique of Faiz & Weatherley (1982) with that used in this study. This would involve engineering the root chamber to enable squeezing of the soil when $E$ was high.

For the undisturbed clay-loam soil, the model, assuming various combinations of $D$, $L_v$ and that the roots were confined to 50, 75 or 100% of the total soil volume, did not agree with the experimental data; implying that the soil hydraulic properties were not inducing a large resistance to flow. Two key observations from the undisturbed clay loam were; (1) few or no roots were visible at the base of the pot for the first and second replicates respectively, suggesting that the roots were confined to the top of the pot and (2) the shape of $B(E)$ for the falling phase of $E$, was usually curved downwards, thereby suggesting that substantial gradients close to the root may have developed during the rising phase of $E$ and disappeared during the falling phase. Also, that the falling phase of $E$ often returned to about the same $B$ as the original point of increasing $E$, and that minimal roots were observed at the base of the pot, collectively suggests that water may have moved from lower in the pot towards the top, where the roots were, presumably, concentrated.

This work provides evidence that low $D(\theta)$ and $L_v$ were not the main resistance to the extraction of water from the undisturbed soil by the plant roots. The clear disagreement between the experimental data and the model has demonstrated that the flow of water to the roots, as encapsulated in the radial-flow model, was not inducing large gradients in suction near the roots in the undisturbed soil. In the undisturbed soil the roots may be
confined to sparsely distributed macropores (Kirkegaard & Lilley 2007; Passioura 1988; Passioura 1991), so that the water takes a long time to reach the roots. The roots may only have partial contact with the wall of the macropore (Watt, Kirkegaard & Rebetzke 2005), so that when compared to a root in full contact, the area for flow was reduced and the suction gradient required to maintain a given flow rate was increased. As low $D(\theta)$ and $L_v$ were not the main resistance to the extraction of water from the undisturbed soil by the plant roots, empirical models that do not rely on measured $L_v$ (Dardanelli et al. 2004) may be of more practical use.
Chapter 6 General Discussion

The extraction of water from the subsoil by wheat crops during grain-filling is especially valuable to grain yield (Kirkegaard et al. 2007; Manschadi et al. 2006; Angus & Herwaarden 2001). However it is hypothesized that water extraction from the subsoil by plant roots may be limited by several factors; a combination of the soil hydraulic properties and low root length density, (unit length of root per unit volume of soil) \( L_v \), an interfacial resistance between the root and the soil, the hydraulic resistance of the plant and the development of an osmotic choke at the root surface (Chapter 1).

This study has explored the hypothesis that the soil is the main resistance to the extraction of water by the plant roots, owing to a combination of low \( L_v \) and low soil water diffusivity, \( D(\theta) \), at low soil water content, \( \theta \). Measurements of the plant transpiration rate, \( E \), and the null measurement of the xylem water potential, \( B \), were made, simultaneously and in real time, on wheat plants grown in undisturbed and repacked clay-loam and repacked sand. The experimental measurements were compared to a model that solves the radial diffusion equation for the flow of water to a single plant root.

An important function for modelling the flow of water to plant roots is \( D(\theta) \) in the suction range from 100 to 1500 kPa. A method was developed for measuring \( D(\theta) \), in that suction range, on undisturbed soil (Chapter 2). This method involved deconvolving the loss of water by evaporation from a soil core in a turbulent atmosphere through time.

In the range from 100 to 1500 kPa suction, \( D(\theta) \) measured (Chapter 3) on the undisturbed clay-loam soil ranged from \( 8 \times 10^{-9} \) to \( 30 \times 10^{-9} \) [m\(^2\) s\(^{-1}\)]. When the undisturbed soil was sieved to less than 2 mm and repacked to the original bulk density of 1.6 g cm\(^{-3}\), \( D(\theta) \) ranged from \( 8 \times 10^{-9} \) to \( 40 \times 10^{-9} \) [m\(^2\) s\(^{-1}\)]; similar to that of the undisturbed soil. The \( D(\theta) \) in the region from 100 to 1500 kPa of the other two soils used for plant water uptake experiments, clay-loam repacked to 1.3 g cm\(^{-3}\) and sand repacked to 1.6 g cm\(^{-3}\), ranged from \( 10 \times 10^{-9} \) to \( 40 \times 10^{-9} \) [m\(^2\) s\(^{-1}\)] and from \( 0.1 \times 10^{-9} \) to \( 50 \times 10^{-9} \) [m\(^2\) s\(^{-1}\)] respectively.
The approximate minimum of the majority of the $D(\theta)$ measurements, in the suction range from 100 to 1500 kPa, was $10 \times 10^{-9}$ [m$^2$ s$^{-1}$]; this is in contrast to the work of Rose (1968), which showed that for most of the soils he used, $D(\theta)$ had a minimum value of approximately $1 \times 10^{-9}$ [m$^2$ s$^{-1}$] at a suction of approximately 1500 kPa. The material with the highest minimum $D(\theta)$ (approximately $4 \times 10^{-9}$ [m$^2$ s$^{-1}$]), in the study of Rose (1968), was described as subsoil clay. It is possible that soil with appreciable clay content, like the undisturbed and repacked clay-loam soil used in this study, has a greater minimum $D(\theta)$.

Published measurements of $D(\theta)$ made on undisturbed soil predominantly involved suctions wetter than 100 kPa. No explicit measurements of $D(\theta)$ were found in the literature in the suction range from 100 to 1500 kPa, the range of interest for modelling the flow of water to plant roots. This study presents both; (1) a novel method, the evaporation method, for measuring $D(\theta)$ in the suction range from 100 to 1500 kPa and (2) novel measurements of $D(\theta)$ on undisturbed soil in the suction range from 100 to 1500 kPa.

The results of the plant water uptake experiments (Chapter 5), for all three soil types, show that during the rising $E$ phase, $B(E)$ was linear at low to moderate $E$. The constant slope of this line, when the difference in suction between the root surface and the bulk soil, $\Delta \tau$, was negligibly small, represented the hydraulic resistance of the plant, $R_{\text{Plant}}$. At high $E$, $B$ often became unstable through time, and accelerated rapidly. For the repacked clay-loam and the repacked sand, this acceleration was accompanied by a falling $E$, as the stomates began to close. During the experimentally controlled falling $E$ phase, for the repacked clay-loam and the repacked sand, $B(E)$ was essentially linear over the whole range of $E$. This was in contrast to the undisturbed clay-loam, where $B$ was often curved downwards with decreasing $E$.

The radial-flow model could match the experimental data, from the repacked clay-loam and the repacked sand, during the rising phase of $E$, if it was assumed that only 10% of the roots were taking up water and if $D$ was constant and low. However it could not match the experimental data during the falling phase of $E$ unless it was assumed that there
had been a significant rise in the hydraulic resistance of the plant, or more likely, that an 
additional interfacial resistance had developed when $E$ was high and $B$ was rapidly 
increasing. The postulated interfacial resistance was about the same size as that within the 
plant, for the repacked sand, and about a quarter of that within the plant for the repacked 
clay-loam.

That the slope of $B(E)$ during the falling phase of $E$, for the repacked clay-loam and the 
repacked sand, was essentially constant suggests that the radial flow of water through the 
soil to the plant roots generated only minor gradients in soil suction. That is the flow of 
water towards the roots was not meeting a large hydraulic resistance. The results on the 
repacked soil show that the putative additional constant resistance developed during the 
time that $B$ was rapidly increasing. That the additional resistance was constant contrasts 
with other discussions of interfacial resistance, where the resistance was effectively a 
function of the soil water content (Bristow, Campbell & Calissendorff 1984; Herkelrath, 
Miller & Gardner 1977b). The additional resistance, encountered in the study reported in 
this thesis, may have been caused by the roots shrinking when $E$ was large, but not 
rehydrating, or only partially so, when $E$ was progressively reduced. This possibility 
accords with Faiz & Weatherley’s (1982) finding that, under high transpiration, 
squeezing or vibrating the soil in which sunflower was growing temporarily raised the 
plant’s water potential. North & Nobel (1997a) also found that shaking the soil 
containing the roots of a water stressed desert succulent facilitated plant water uptake. 
Huck, Klepper & Taylor (1970) observed that the diameter of a cotton root shrunk to 
60% of its maximum under high evaporative demand. They observed a diurnal pattern 
where the root shrunk under high evaporative demand during the day and then increased 
in diameter overnight back to the original diameter.

The hypothesis that root shrinkage occurs at high $E$ and that the root diameter only partly 
recovers when $E$ is decreased, resulting in a step-change increase in the apparent 
resistance to flow within the system, could be tested by combining the experimental 
technique of Faiz & Weatherley (1982) with that used in this study. This would involve 
engineering the root chamber to enable squeezing of the soil when $E$ was high.
The radial-flow model did not agree with the experimental data from the undisturbed clay-loam soil, even when various combinations of a wide range of values of $D$ and $L_v$ were tried. This disagreement may have been due to a skewed distribution of roots in the cores, for few if any roots could be seen at the base of any core. However, even when the roots were assumed to be confined to the top 50 or 75% of the total soil volume, the model and the experimental data did not agree.

Nevertheless, a downward curving shape of $B(E)$ for the falling phase of $E$ was evident in most of the undisturbed clay-loam experiments, thereby suggesting that substantial gradients close to the root may have developed during the rising phase of $E$, and may have collapsed during the falling phase. Alternatively, because the falling phase of $E$ often returned $B$ to about the same point at which it started, and because there were few roots in the lower part of the core, water may have moved from lower in the pot towards the top, thereby reducing the soil water suction where the bulk of the roots were.

This work provides evidence that the flow of water to roots, as encapsulated in the radial-flow model, does not induce large gradients in suction near the plant roots growing in the three soil types used in the experiments reported here. The clear disagreement between the experimental data and the model suggests that something else was generating the large hydraulic resistances evident between the soil and the leaves of the plants.

In the repacked soils, irreversible shrinkage of the roots is a possible explanation for the putative additional constant resistance that developed during the time that $B$ was rapidly increasing. The experimental work of Huck, Klepper & Taylor (1970) showed that roots can take approximately four to six hours to recover their original diameter after root shrinkage; this result suggests that if root shrinkage did occur, in the study reported in this thesis, then there would not have been enough time for the roots to recover their original diameter. The hypothesis that root shrinkage does recover overnight could be investigated by leaving the plant overnight and repeating the experiment the following morning and observing the slope of the rising $E$ phase.

In the undisturbed soil, the roots may be clumped in sparsely distributed macropores (Kirkegaard & Lilley 2007; Passioura 1988; Passioura 1991), in such conditions the water
takes a long time to reach the root. The roots may only have partial contact with the wall of the macropores (Watt, Kirkegaard & Rebetzke 2005), so that when compared to a root in full contact, the area for flow is reduced and the suction gradient required to maintain a given flow rate is increased. Given that this work provides evidence that low $D(\theta)$ and $L_v$ were not the main resistance to the extraction of water from the undisturbed soil by the plant roots, empirical models that do not rely on measured $L_v$ (Dardanelli et al. 2004) may be of more practical use.

Future work could be focused on determining the length of root that is active in water extraction, thereby providing more information for testing the model. The plant water uptake experimental technique could be improved by including soil moisture instrumentation to determine the temporal and spatial distribution of the soil water content. Knowledge of the spatial and temporal changes in soil water content would provide an indirect estimation of root activity and permit rigorous testing of the model.
References


Baughn, JW & Tanner, CB 1976, 'Leaf Water Potential - Comparison of Pressure Chamber and Insitu Hygrometer on 5 Herbaceous Species', *Crop Science*, vol. 16, no. 2, pp. 181-184.


Crescimanno, G & Iovino, M 1995, 'Parameter-Estimation by Inverse Method Based on One-Step and Multistep Outflow Experiments', *Geoderma*, vol. 68, no. 4, pp. 257-277.


French, RJ & Schultz, JE 1984, 'Water-Use Efficiency of Wheat in a Mediterranean-Type Environment .2. Some Limitations to Efficiency', *Australian Journal of Agricultural


Hoagland, DR & Arnon, DI 1950, 'The water culture method for growing plants without soil', *California Agricultural Experiment Station Circular*, vol. 347, pp. 1-32.


Simunek, J, Sejna, M & van Genuchten, MT 1999, *The HYDRUS-2D software package for simulating the two-dimensional movement of water, heat and multiple solutes in variably-saturated media*, U.S. Salinity Laboratory Agricultural Research Service and U.S. Department of Agriculture, Riverside, California, USA.


van Genuchten, MTh 1987, *A numerical model for water and solute movement in and below the root zone*, US Salinity Laboratory, USDA, ARS, Riverside, California.


Watt, M, Kirkegaard, JA & Rebetzke, GJ 2005, 'A Wheat Genotype Developed for Rapid Leaf Growth Copes Well With the Physical and Biological Constraints of Unploughed Soil',

Appendix 1

Plots of temperature depression, due to evaporation, at the soil surface measured by thermocouple as a function of time for clay-loam soil repacked to a bulk density of 1.3 and 1.6 [g cm$^{-3}$] and sand repacked to a bulk density of 1.6 [g cm$^{-3}$]. The soil surface was subjected to the airstream at $t = 0$. 

![Soil surface temperature plots](image)
Appendix 2

Plot model error for Figure 11(1a) when; (1) quadratic and exponential $D(\theta)$ functions were optimized over the total time (total time) and (2) when quadratic and exponential $D(\theta)$ functions were optimized using evaporation data from the first $10^5$ s, at which point the mean $\theta$ in the core is equal to 1500 kPa (1500 kPa time).
Appendix 3

Derivation of radial diffusion equation

The diffusion equation is cast in radial coordinates by firstly considering the idealized model of a plant root; that is a slice of a cylindrical shell, where \( r \) is the radius of the root and \((r + \Delta r)\) is the radius of the outer boundary of soil that the root is assumed to have exclusive access to. The mass balance for the shell is:

\[
\text{(Flow in at r) - (Flow out at r + \Delta r)} = \text{Rate of gain between r and r + \Delta r} \tag{66}
\]

The flow in at \( r \) can be expressed as:

\[
\text{Flow in at } r = \text{Area} \times \text{Velocity} \times \text{Time} \tag{67}
\]

\[ q_r = [2\pi h]u(r)\Delta t \]

Where \( u \) is velocity and \( h \) is height. Similarly, the flow out at \((r + \Delta r)\) can be expressed as:

\[
\text{Flow out at } r + \Delta r = \text{Area} \times \text{Velocity} \times \text{Time} \tag{68}
\]

\[ q_{r + \Delta r} = [2\pi (r + \Delta r)h]u(r + \Delta r)\Delta t \]

The rate of gain in the shell volume is:

\[
\text{Rate of gain} = (\theta \times \text{Volume})_{r + \Delta r} - (\theta \times \text{Volume})_r \tag{69}
\]

\[ = \theta \pi \left[(r + \Delta r)^2 - r^2\right]h_{r + \Delta r} - \theta \pi \left[(r + \Delta r)^2 - r^2\right]h_r \]

Subbing eq. 67, eq. 68 and eq. 69 into eq. 66,

\[
[2\pi h]u(r)\Delta t - [2\pi (r + \Delta r)h]u(r + \Delta r)\Delta t = \theta \pi \left[(r + \Delta r)^2 - r^2\right]h_{r + \Delta r} - \theta \pi \left[(r + \Delta r)^2 - r^2\right]h_r \tag{70}
\]

expanding,

\[
(r + \Delta r)^2 - r^2 = (r + \Delta r)(r + \Delta r) - r^2 \]

\[ = r^2 + r\Delta r + r\Delta r + \Delta r^2 - r^2 \]

\[ = 2r\Delta r + \Delta r^2 \]

and dividing by \( 2\pi \Delta r h \Delta t \),

\[
\frac{ru(r) - (r + \Delta r)u(r + \Delta r)}{\Delta r} = \theta \frac{(r + \Delta r/2)_{r + \Delta r} - \theta(r + \Delta r/2)_r}{\Delta t} \tag{72}
\]

then taking the limit as \( \Delta r \) and \( \Delta t \) go to zero.

\[
-\frac{\partial}{\partial r}[ru(r)] = r \frac{\partial \theta}{\partial t} \tag{73}
\]
Eq. 73 is the conservation equation in radial coordinates. If we then define velocity as a function of radial distance in terms of soil water diffusivity,

\[ u(r) = D(\theta) \frac{\partial \theta}{\partial r} \]  

(74)
eq. 73 becomes the radial diffusion equation.

\[ \frac{\partial \theta}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left[ rD(\theta) \frac{\partial \theta}{\partial r} \right] \]  

(75)

After expansion:

\[ \frac{\partial \theta}{\partial t} = \frac{1}{r} D(\theta) \frac{\partial \theta}{\partial r} + \frac{\partial}{\partial r} \left[ D(\theta) \frac{\partial \theta}{\partial r} \right] \]  

(76)

With the initial and boundary conditions:

\[ t = 0, \quad r_a \leq r \leq r_b, \quad \theta = \theta_i \]

\[ t > 0, \quad r = r_a, \quad D(\theta) \frac{\partial \theta}{\partial r} 2\pi r_a h = \frac{Q(r_b^2 - r_a^2)}{2r_a} \pi h \]  

(77)

\[ t > 0, \quad r = r_b, \quad \frac{\partial \theta}{\partial r} = 0 \]

Eq. 75 is solved numerically by setting

\[ R = \ln(r/r_a) \]

\[ e^R = r/r_a \]

\[ r = r_a e^R \]  

(78)

\[ \begin{align*} \frac{\partial r}{\partial R} &= r_a e^R \quad \text{when } R = 0, \frac{\partial r}{\partial R} = r_a \\ \frac{\partial r}{\partial R} &= r_a e^R \frac{\partial R}{\partial R} \end{align*} \]

the log transform reduces the number of distance steps needed in the finite difference grid, especially if \( r/r_a \) is large (Passioura & Cowan 1968). Subbing eq. 78 into eq. 75,

\[ \frac{\partial \theta}{\partial t} = \frac{e^{-2R}}{r_a^2} \left( \frac{\partial}{\partial R} \left( D \frac{\partial \theta}{\partial R} \right) \right) \]  

(79)

after expansion:

\[ \frac{\partial \theta}{\partial t} = \frac{e^{-2R}}{r_a^2} \left( \frac{\partial D}{\partial R} \frac{\partial \theta}{\partial R} + D \frac{\partial^2 \theta}{\partial R^2} \right) \]  

(80)

With the initial and boundary conditions:

\[ t = 0, \quad 0 \leq \ln(r_b/r_a) \leq R_b, \quad \theta = \theta_i \]

\[ t > 0, \quad R = 0, \quad D(\theta) \frac{\partial \theta}{r_a e^R} 2\pi r_a h = \frac{Q(r_b^2 - r_a^2)}{2r_a} \pi h \]  

(81)

\[ t > 0, \quad R = R_b, \quad \frac{\partial \theta}{\partial R} = 0 \]
We now express eq. 80 in finite difference form.

\[
\frac{\partial D}{\partial R} \frac{\partial \theta}{\partial R}\bigg|_{i,j} = \left( \frac{D_{i+1,j} - D_{i-1,j}}{2R} \right) \frac{\theta_{i+1,j} - \theta_{i-1,j}}{2R} = \frac{1}{4} \left( \frac{D_{i+1,j} - D_{i-1,j}}{2R} \right) \left( \theta_{i+1,j} - \theta_{i-1,j} \right) \tag{82}
\]

\[
\frac{D}{\partial R^2} \frac{\partial^2 \theta}{\partial R^2}\bigg|_{i,j} = D_i \left[ \frac{\partial \theta}{\partial R}\bigg|_{i+1/2} - \frac{\partial \theta}{\partial R}\bigg|_{i-1/2} \right] - \frac{1}{\partial R} \left[ \frac{\theta_{i+1,j} - \theta_{i-1,j} - \theta_{i,j} - \theta_{i-1,j}}{\partial R} \right] = \frac{1}{\partial R} \left[ \theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j} \right] \tag{83}
\]

The equation to be solved is then:

\[
\frac{\theta_{i,j} - \theta_{i,j+1}}{\partial t} = \frac{e^{-2R}}{r_a^2} \frac{1}{\partial R^2} \left[ \left( \frac{D_{i+1,j} - D_{i-1,j}}{4} \right) \left( \theta_{i+1,j} - \theta_{i-1,j} \right) + D_i \left( \theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j} \right) \right] = \frac{e^{-2R}}{r_a^2} \left[ \frac{1}{\partial R^2} \left( \frac{D_{i+1,j} - D_{i-1,j}}{4} \right) \left( \theta_{i+1,j} - \theta_{i-1,j} \right) + D_i \left( \theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j} \right) \right] \tag{84}
\]

The derivative at the three space coordinates on the \( j \) time level are then used to form a Jacobian matrix. The system of linear equations is then solved using the Thomas algorithm. The derivatives at \( \left( \theta_{i-1,j}, \theta_{i,j}, \theta_{i+1,j} \right) \) are:
\[
\partial \theta_{i-1,j} = \frac{e^{-2R}}{r_a^2} \frac{1}{\partial R^2} \left[ -\frac{(D_{r+1,j} - D_{r-1,j})}{4} + D_i \right]
\]
\[
\partial \theta_{i,j} = \frac{e^{-2R}}{r_a^2} \frac{1}{\partial R^2} \left[ D_i(-2) \right] - \frac{1}{\partial t} \left[ \frac{1}{\partial t} \right]
\]
\[
\partial \theta_{r+1,j} = \frac{e^{-2R}}{r_a^2} \frac{1}{\partial R^2} \left[ \frac{(D_{r+1,j} - D_{r-1,j})}{4} + D_i \right]
\]

(85)
Appendix 4

Measurements of B and E: first replicate of undisturbed clay-loam soil

Measurements of $B$ and $E$ were made on the first replicate (initially equilibrated at 10 kPa suction) of undisturbed clay-loam soil on days 23, 24, 28 (a root was seen at the base of the core on day 27) and 29, at which point the plant had dried the soil to the extent that the xylem sap could no longer be raised to atmospheric pressure, without exceeding the 2 MPa pressure limit of the equipment. The slope and intercept from the first replicate of undisturbed clay-loam, for both increasing and decreasing $E$, and the leaf area are shown in Table 11 and the data is plotted in Figure 56.

Table 11 First replicate of undisturbed clay-loam soil: slope, intercept and leaf area for each of the $B$ and $E$ experiments shown in Figure 56. ** Denotes that the plant was left in the chamber overnight and therefore the leaf area was not measured on day 24. The intercept and slope for decreasing $E$ could not be estimated on days 28 and 29 due to the non-linear shape of $B(E)$.

<table>
<thead>
<tr>
<th>Age (days from sowing)</th>
<th>Intercept (up) [kPa]</th>
<th>Slope (up) [kPa s $\mu g^{-1}$]</th>
<th>Intercept (down) [kPa]</th>
<th>Slope (down) [kPa s $\mu g^{-1}$]</th>
<th>Leaf area [cm$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>23</td>
<td>253</td>
<td>2.8</td>
<td>479</td>
<td>2.2</td>
<td>59.0</td>
</tr>
<tr>
<td>24</td>
<td>340</td>
<td>2.5</td>
<td>692</td>
<td>1.9</td>
<td>**</td>
</tr>
<tr>
<td>28</td>
<td>810</td>
<td>3.1</td>
<td>NA</td>
<td>NA</td>
<td>95.0</td>
</tr>
<tr>
<td>29</td>
<td>1060</td>
<td>2.9</td>
<td>NA</td>
<td>NA</td>
<td>105.0</td>
</tr>
</tbody>
</table>
Figure 56 First replicate of undisturbed clay-loam soil: B as a function of E measured on four different days.
**Measurements of B and E: second replicate of undisturbed clay-loam soil**

![Graphs showing transpiration rate (E) as a function of balancing pressure (B)](image)

Figure 57 Second replicate of undisturbed clay-loam soil: B as a function of E. The plant was re-watered back to 20 kPa weight after the experiment on days 30 and 36.
Measurements of $B$ and $E$: First replicate of repacked clay-loam soil

Table 12 First replicate of repacked clay-loam soil. Slope, intercept and leaf area for each of the $B$ and $E$ experiments are shown in Figure 58 and Figure 59. Note that the experiment was aborted on day 38 when the pressure seal failed.

<table>
<thead>
<tr>
<th>Age (days from sowing)</th>
<th>Intercept (up) [kPa]</th>
<th>Slope (up) [kPa s $\mu g^{-1}$]</th>
<th>Intercept (down) [kPa]</th>
<th>Slope (down) [kPa s $\mu g^{-1}$]</th>
<th>Leaf area [cm$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>71</td>
<td>3.2</td>
<td>83</td>
<td>3.1</td>
<td>17.4</td>
</tr>
<tr>
<td>27</td>
<td>92</td>
<td>2.2</td>
<td>169</td>
<td>1.7</td>
<td>24.0</td>
</tr>
<tr>
<td>30</td>
<td>119</td>
<td>2.0</td>
<td>149</td>
<td>1.9</td>
<td>32.6</td>
</tr>
<tr>
<td>32</td>
<td>80</td>
<td>2.0</td>
<td>177</td>
<td>1.7</td>
<td>40.6</td>
</tr>
<tr>
<td>34</td>
<td>133</td>
<td>1.8</td>
<td>193</td>
<td>1.6</td>
<td>51.0</td>
</tr>
<tr>
<td>36</td>
<td>169</td>
<td>1.8</td>
<td>323</td>
<td>1.7</td>
<td>59.7</td>
</tr>
<tr>
<td>38</td>
<td>331</td>
<td>2.3</td>
<td>NA</td>
<td>NA</td>
<td>75.7</td>
</tr>
</tbody>
</table>

Figure 58 First replicate wheat plant grown in repacked clay-loam soil: $B$ as a function of $E$ for days 24 to 32.
Figure 59 First replicate wheat plant grown in re-packed clay-loam soil: $B$ as a function of $E$ for days 34 to 38. Note the experiment was aborted on day 38 when the pressure seal failed.

**Measurements of $B$ and $E$: Second replicate of re-packed clay-loam soil**

Table 13 Second replicate of re-packed clay-loam soil. Slope, intercept and leaf area for each of the $B$ and $E$ experiments shown in Figure 60 and Figure 61. Plant was re-watered back to the 10 kPa suction weight after experiments on days 27 and 37. Note that the experiment was aborted on day 43 when the leaf sensor failed.

<table>
<thead>
<tr>
<th>Age (days from sowing)</th>
<th>Intercept (up) [kPa]</th>
<th>Intercept (down) [kPa]</th>
<th>Slope (up) [kPa s $\mu$g$^{-1}$]</th>
<th>Slope (down) [kPa s $\mu$g$^{-1}$]</th>
<th>Leaf area [cm$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>23</td>
<td>261</td>
<td>381</td>
<td>1.8</td>
<td>1.5</td>
<td>41.9</td>
</tr>
<tr>
<td>25</td>
<td>358</td>
<td>562</td>
<td>2.0</td>
<td>2.0</td>
<td>54.2</td>
</tr>
<tr>
<td>27</td>
<td>424</td>
<td>658</td>
<td>2.8</td>
<td>3.1</td>
<td>64.5</td>
</tr>
</tbody>
</table>
Figure 60 Second replicate wheat plant grown in repacked clay-loam soil: $B$ as a function of $E$ for days 23 to 30. Plant was re-watered back to the 10 kPa suction weight after experiments on days 27 and 37.
Figure 61 Second replicate wheat plant grown in repacked clay-loam soil: $B$ as a function of $E$ for days 33 to 43. Plant was re-watered back to the 10 kPa suction weight after experiments on days 27 and 37. Note the experiment was aborted on day 43 when the leaf sensor failed.
Measurements of B and E: Third replicate of repacked clay-loam soil

Figure 62 Third replicate wheat plant grown in repacked clay-loam soil: B as a function of E for days 22 to 32.
Measurements of $B$ and $E$: First replicate of repacked sand

Table 14 First replicate of repacked sand: slope, intercept and leaf area for each of the $B$ and $E$ experiments shown in Figure 63 and Figure 64. Slope and intercept for increasing and decreasing $E$ on day 23 was not calculated because data looked unreliable.

<table>
<thead>
<tr>
<th>Age (days from sowing)</th>
<th>Intercept (up) [kPa]</th>
<th>Slope (up) [kPa s $\mu g^{-1}$]</th>
<th>Intercept (down) [kPa]</th>
<th>Slope (down) [kPa s $\mu g^{-1}$]</th>
<th>Leaf Area [cm$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>23</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>21.3</td>
</tr>
<tr>
<td>25</td>
<td>65</td>
<td>4.3</td>
<td>212</td>
<td>5.5</td>
<td>26.5</td>
</tr>
<tr>
<td>27</td>
<td>65</td>
<td>5.8</td>
<td>133</td>
<td>7.6</td>
<td>31.1</td>
</tr>
<tr>
<td>28</td>
<td>-91</td>
<td>5.5</td>
<td>51</td>
<td>6.7</td>
<td>33.6</td>
</tr>
<tr>
<td>29</td>
<td>37</td>
<td>3.8</td>
<td>66</td>
<td>6.0</td>
<td>36.5</td>
</tr>
<tr>
<td>31</td>
<td>71</td>
<td>3.4</td>
<td>280</td>
<td>6.7</td>
<td>43.2</td>
</tr>
<tr>
<td>33</td>
<td>87</td>
<td>4.3</td>
<td>190</td>
<td>8.7</td>
<td>51.8</td>
</tr>
<tr>
<td>35</td>
<td>145</td>
<td>5.0</td>
<td>217</td>
<td>9.6</td>
<td>59.6</td>
</tr>
<tr>
<td>37</td>
<td>307</td>
<td>5.6</td>
<td>482</td>
<td>6.6</td>
<td>65.3</td>
</tr>
<tr>
<td>39</td>
<td>416</td>
<td>6.6</td>
<td>709</td>
<td>6.4</td>
<td>71.6</td>
</tr>
</tbody>
</table>
Figure 63 First replicate of repacked sand: $B$ as a function of $E$ for experiments made on the days 23 to 31.
Figure 64 First replicate of repacked sand: $B$ as a function of $E$ for experiments made on days 33 to 39.
Measurements of $B$ and $E$: Second replicate of repacked sand

Figure 65 Second replicate of repacked sand: $B$ as a function of $E$, prior to and including day 29.